

**EVALUATING THE SIGNIFICANCE AND
ADVANCEMENT OF THREE-DIMENSIONAL
ISLAMIC ARCHITECTURAL GEOMETRIC
PATTERNS: MUQARNAS**

Master of Science Thesis

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FINAL APPROVAL FOR THESIS

This thesis titled “Evaluating the Significance and Advancement of Three-Dimensional Islamic Architectural Geometric Patterns: Muqarnas” has been prepared and submitted by Samia Ibrahim AHMED in partial fulfillment of the requirements in “Anadolu University Directive of Graduate Education and Examination” for the Degree of Master of Science (M.Sc.) in Architecture Department has been examined and approved on 14/10/2019.

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ABSTRACT

EVALUATING THE SIGNIFICANCE AND ADVANCEMENT OF THREE-DIMENSIONAL ISLAMIC ARCHITECTURAL GEOMETRIC PATTERNS: MUQARNAS

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Program in Building Design

Anadolu University, Graduate School of Science, June 2019

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Geometry is a formative idea for art and architecture. Focusing on Islamic architecture as the topic of this research, it can be said that Muslims had a complex understanding of Non-Euclidean geometry shown in their arts and architecture. Geometric patterns have a great significance in Islamic architecture. The thesis starts with the aim of research and continues with a chapter about the mathematical aspects of geometry and patterns giving a technical view on the principles of their application in art and architecture. Geometric patterns are the most consistent part of Islamic architecture all over the world how they have been adapted and changed or developed through time and technology to contemporary Islamic architecture shows the development level and complexity of the Islamic world in modern days. The evolution and advancement of two and three-dimensional Islamic geometric patterns are reflected focusing on Muqarnas. The continuity of Islamic geometric patterns especially Muqarnas as a lively theory that is not only copy and passed from previous works but can be advanced with technology to integrate it in contemporary works is the aim of this research. The way to do that is by studying it in pieces or parts to be developed rather than a complete object by itself.

Keywords: Geometry, Islamic Geometric patterns, 2D Islamic geometric patterns, Muqarnas

ÖZET

ÜÇ BOYUTLU İSLAM MİMARİSİ GEOMETRİK DESENLERİNİN ÖNEMİ VE GELİŞTİRİLMESİNİN DEĞERLENDİRİLMESİ: MUKARNAS

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Geometri, sanat ve mimariyi biçimlendiren bir fikirdir. Bu araştırmada İslam mimarisine odaklanarak, Müslüman sanatı ve mimarisinde gösterilen karmaşık bir anlayışa sahip olan Öklid Dışı geometrinin incelenmesi amaçlanmıştır. İslam mimarisinde geometrik desenler önemli başlıklardandır. Tezin başlığında araştırma amacıyla alakalı bilgi verilerek devam eden bölümde ise sanat ve mimarlıkta uygulama ilkeleri hakkında teknik bir görüş veren geometri ve kalıpların matematiksel yönleri hakkında bir inceleme sunulmuştur. Geometrik desenler, zaman içinde ve çağdaş İslam mimarisinde teknoloji ile adapte edilen, değiştirilen veya geliştirilen tam anlamıyla İslam mimarisinde bir tutarlılık içerisinde devam eden bir alandır. İki ve üç boyutlu İslami geometrik kalıpların evrimi ve ilerlemesi Mukarnas'a odaklanarak gelişmiştir. İslami geometrik kalıpların devamlılığının sağlanması, özellikle Mukarnas'tan ve önceki geometrik çalışmalardan desenlerin aktarılmasıyla, çağdaş çalışmalara entegre edilecek teknolojiyle geliştirilebilecek canlı bir teori olarak kullanılması bu araştırmanın amacını oluşturmaktadır. Bir metod olarak, kendisi tarafından tam bir nesneden ziyade Mukarnas'tan geliştirilecek parçalar halinde üzerine çalışılması sunulmuştur.

Anahtar Sözcükler: Geometri, Geometrik desenler, 2D İslami geometrik desenler, Mukarnas

STATEMENT OF COMPLIANCE WITH ETHICAL PRINCIPLES AND RULES

I hereby truthfully declare that this thesis is an original work prepared by me; that I have behaved in accordance with the scientific ethical principles and rules throughout the stages of preparation, data collection, analysis and presentation of my work; that I have cited the sources of all the data and information that could be obtained within the scope of this study, and included these sources in the references section; and that this study has been scanned for plagiarism with “scientific plagiarism detection program” used by Anadolu University, and that “it does not have any plagiarism” whatsoever. I also declare that, if a case contrary to my declaration is detected in my work at any time, I hereby express my consent to all the ethical and legal consequences that are involved.

.....
Samia Ibrahim AHMED

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The ultimate Praise and Thanks due to Allah, before and after.

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ABBREVIATION INDEX

IGP	: Islamic Geometric Patterns
IA	: Islamic Architecture
NEG	: Non- Euclidean Geometry
2D	: Two-Dimension
3D	: Three-Dimension
2DPP	: Two-Dimensional Pattern Plans
CE	: Current Era
i.e.	: That is
<i>Tawhid</i>	: is the indivisible oneness concept of monotheism in Islam

1. INTRODUCTION

Geometry has been central element since the existence of the world and man started researching nature to understand it. Geometry is valuable in the universe for each of its parts. It is by points, lines, angles and geometric figures that nature is understood and expressed. It is a formative idea and design tool that is used since the mankind started expressing himself in art and architecture or construct buildings. But Muslims from 9th and 10th century especially, introduced a further complex and unique application of geometry and patterns in art and architecture to the world.

The geometric patterns of Islamic art arise from the Islamic view of the world. The representation of animals and people are generally forbidden, which explains the preference for abstract geometric patterns. The concept of *Tawhid*; ‘unity in multiplicity and multiplicity in unity’, which is also expressed in Islamic art and architecture especially in Islamic Geometric Patterns.

The paper starts with definition of geometry, historical background and application of it in human life. There are different types of geometries in math. However, in this paper geometry related with architecture is discussed. Patterns are also reflection of the study method of geometry of nature. Mainly focuses on geometric patterns in Islamic architecture (Islamic geometric patterns) concentrating on three-dimensional geometric patterns, by the other name Muqarnas.

The aim and question of this paper is on defining this amazing innovation of Islamic architecture of the 9th and 10th century which is unique and original work of Muslims at the time. The advancement of two-dimensional and three-dimensional patterns has continued up to 16th century. Later, it has not developed or become a center piece to be integrated with technology being part of the contemporary designs and astonish the world again with its art and architecture. Therefore, studying it in different method on how to make it as a progressive design theory rather than a finished object in order to give room for new innovations.

The evolution, and advancement of geometric patterns in Islamic architecture is studied as two- and three- dimension in this research paper.

The advancement of Islamic geometric patterns is studied as qualitative data to its maximum innovation or pick point and taken into part and elements to understand it as theory or principle with its main defining elements.

The elements are then taken into consideration with basic principles of design to transform it to new design principles with the help of technology and software programs in two divisions as self-generated and program assisted new designs. Examples with methods of new design principle and samples are shown as a start. Better works and detail researches can be done to continue the idea and integrate muqarnas in modern Islamic art and architecture.

1.1. Problem Statement

Earlier Islamic art specifically Islamic geometric patterns always had a mollifying and charming effect. The geometric figures were not created only to please the eye, but also to address the human spirits. Geometric patterns and Islamic art and architecture are inseparable topics where the mentioning of one recalls the other since they are the common characters of Islamic art and architecture in different parts of the world. The evolvment and change through time are of this Islamic geometric pattern is always the question.

The thesis is a research to answer the questions of definition of Islamic geometric patterns, especially Muqarnas, development and progress up to 14th century and through time with the advancement of technology and digitalized world how they can be a progressing theory that has the tendency for new innovations rather than departed rigid objects. Since, not much has been done to develop them to become this century's center piece in contemporary works as it was a great innovation in the 14th century.

1.2. Research Objective

Geometric patterns are common language used in Islamic art and architecture all over. Muqarnas are the highest innovations in Islamic architecture, which also shows the advancement of Muslims and/or Islamic world in mathematics, physics, philosophy and imagination or representation of ideal things into space and tangible art piece of work that can be constructed and realized. Not to lose this important piece, researches and developments have to be made so it can develop with technology, and innovation methods or principles must be appointed for it to be alive and integrated in the contemporary works.

1.3. Scope of Research

The research is limited to the geometric patterns of Islamic architecture in two-dimension and three-dimension, especially of Muqarnas from the ocean of Islamic art and architecture. Other major pieces of Islamic art and architecture as flora or vegetal and calligraphy works are not included in the research. Since geometric pattern is considered the most abstract form of Islamic art that contributed to shift the focus from direct representation of things (such as human or animal figures) as they appear to complex mathematical based illustration.

In terms of time frame since the aim of the research is to find the pieces and defining characters of muqarnas, selected examples and all the possible varieties or types are considered from 11th – 14th century, where the advancement of muqarnas was at its pick. Periods in the Islamic dynasty were the art and architecture focus on vegetal or flora and calligraphy bases are not included.

Adding colors or studying Islamic Geometric Patterns with their color patterns is another broad topic to study which is beyond the scope of this thesis, therefore, discarding all colors and interlacing material arriving at what is so called ‘the patterns design’ i.e. a diagram consisting of line segments

1.4. Research Methodology

The research uses primary and secondary data. Qualitative data is gathered about geometric patterns in Islamic architecture specifically of three-dimensional geometric patterns; muqarnas.

In the first phase of the thesis, review of history of geometry and the application in art and architecture followed by geometric patterns was made.

After this analysis, Islamic geometric patterns are reviewed using research done on survived buildings study showing the advancement of IGP in time frame and places. The progress is documented and summarized, as a qualitative data using the elements of composition in Muqarnas evolution. This study and evaluation in digital works determine its potential for new innovative designs. Then final stage being evaluating of these new forms in contemporary muqarnas sample designs and experimentation to be applied in important Islamic buildings.

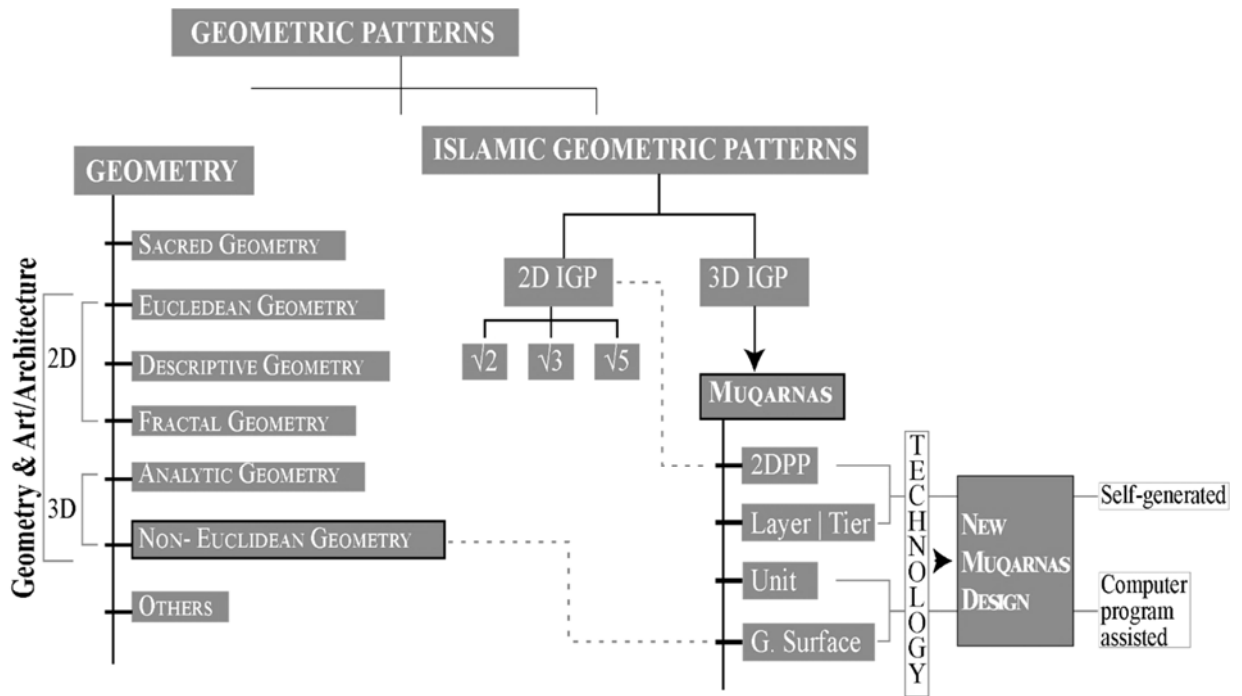


Figure 1.1. Diagram Showing the research relationship and method

1.5. Limitations

There have been great limitations on the study of *Muqarnas* and Islamic architecture in general, as the topic is broad and confusing on exact and common definition of terminologies. Hence, one can hardly find any the necessary materials and field work is impossible due to political instability and continues civil war going on. Moreover, due to these circumstances the heritage buildings and sites are damaged, demolished or not in a good condition to study. Even if there was stabile financial shortage will not allow me to travel to all the research sites as well.

2. GEOMETRY AND PATTERNS

Geometry started from the first-time mankind started living in caves and carving on the walls to decorate and express or represent the lifestyle thought, mark their existence and ownership of the place at some time.

In this chapter geometry and patterns are discussed starting with the definition of the terms followed by the historical background and origin in respect with different theories and finally the significant part of this thesis their relationship and application in architecture.

2.1. Definition of Geometry and Patterns

“It is impossible to know nature without geometry. The principles of geometry have absolute value throughout the universe and for each of its parts. It is by lines, angles, and geometric figures that all-natural phenomena must be understood.” (Necipoğlu, 1995, p. 162)

The above statement clearly describes how much geometry is the center life and this earth both for understanding nature and interfering as a human being in construction or destruction of the earth let along the universe. Human being has been a wondering creation to understanding the purpose of existence the urge of finding an answer led to researches in every field of study starting with understanding nature, form, patterns, geometry and so on. The importance of geometry in human life intellectually is described by Ibn Khaldun as mental sharpening factual capable of cleaning like soap cleans dirt:

“It should be known that geometry enlightens the intellect and sets one’s mind right. All its proofs are very clear and orderly. It is hardly possible for errors to enter into geometric reasoning, because it is well arranged and orderly. Thus, the mind that constantly applies itself to geometry is not likely to fall into error Our teachers used to say that one’s application to geometry does to the mind that soap does to a garment. It washes off strains and cleanses it of grease and dirt.” (Khaldun, 1967, pp. 130-131).

In mathematics the study geometry is mainly classified as: Euclidean geometry, which is referred as true geometry, Elliptical geometry, with a main principle of all lines meet, and Non- Euclidean geometry, that is there are infinite many parallels to a line through each point. In Non- Euclidean Geometry if two parallel lines are bent around a sphere, these lines meet at the top and bottom of the sphere. With this theory comes

Spherical and Hyperbolic geometry. Spherical geometry which is simply defines as, “no line through point P is parallel to line L”, and Hyperbolic geometry, “at least two lines through P are parallel to L”. This new idea brings uncertainties and challenges as “distances and sums of degrees in triangles that people once accepted as fact were now questionable, depending on the type of surface”. (Carter, 1996, p.20)

Geometry has also been applied and continued to be used in architecture starting from when human beings builds shelters and monuments. Through time the use and understanding of geometry become complicated since the studying of patterns and ratios of nature gets deeper.

Pattern is defined in Cambridge dictionary as “any regularly repeated arrangement, especially a design made from repeated lines, shapes, or colors on a surface”. Everything on this world is a composition of patterns and rhythms. Nature has its own pattern objects, sound to details of leaves and flowers are all result of composition of patterns. The Muslim Andalusian philosopher and physician Ibn Rushd (Averroes 1128-1198 C.E.) said about nature pattern order comparing it with art “in product of art, to which they compared the products of nature, there exists order and proportion and this was called wisdom, and they call the creator wise” (Necipoğlu, 1991, p. 103). Following this human beings also created and adapted patterns from the ancient civilization in their arts, architecture, buildings and constructions in general based on different believes and natural adaptations. To understand geometric patterns, it is mandatory to know the basic types of geometry in math that are described in short in the previous page. The origin of the study of geometry and historical background is discussed below focusing on its relationship with architecture.

2.2. Historical Background of Geometry

The word Geometry is originated from ancient Greek word *Geo-* earth, *-Metron* measurement together meaning earth measurement. The study of geometry has thrived for ages. The ancient civilizations of Egypt, Indus and Sumer has studied the empirical side of geometry that was appropriate for their architecture.

“The idea that began its journey with the pyramids of Egypt, about 2600B.C, entered Greece about 2300 years later and ‘transformed geometry into an exact abstract reasoning device. In both Egypt and Mesopotamia geometry had provided not only an instrument of measurement but produced monumental modes of artistic expressions It was only the spirit of

Islam which about 1200 years later transformed the rationality of abstract thinking enshrined within geometry into a sensory form of artistic expression” (Araeen, 2010, p. 509).

The development of Euclidean geometry extends back to about ten thousand years before the Birth of Christ. It was a great discovery and practical value to ancient Greek in designing and constructing buildings, which we still use today as well. Some of the most known elements in definitions of Euclid are the Vitruvian man and Francesco de Giorgio’s inscribed man figures.

Vitruvian man is, Leonardo da Vinci’s drawing of a man figure with starched arms, simultaneously inscribed a circle and a square with notes from the works of the famous architect Vitruvian (Figure 2.1).

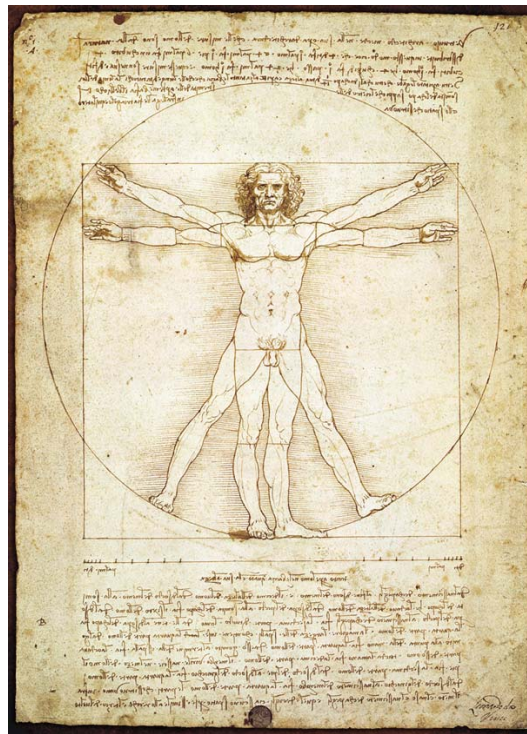


Figure 2.1. *Vitruvian man by Leonardo da Vinci (Lawlor, 1982)*

Francesco di Giorgio’s demonstration of using inscribed man figure to generate a planning system for designing Cathedrals. His drawing of Cathedral man is compatible with Michelangelo’s St. Peter of Rome floor plan and Siena Cathedral of Siena, Italy, which was dedicated to him.

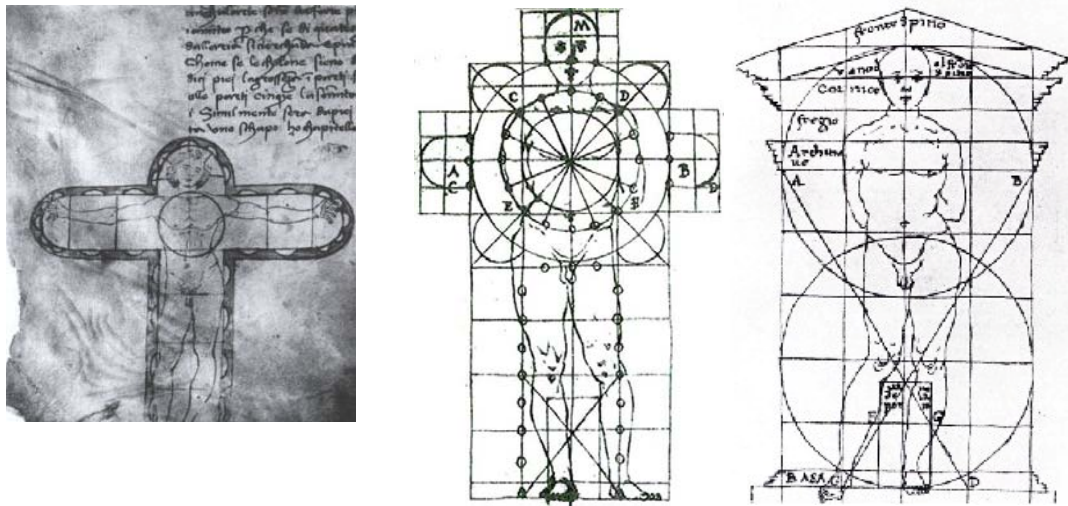


Figure 2.2. Francesco di Giorgio's man figures, (a) direct representation drawing for Cathedral Church, (b) figure with proportions for Church plan, (c) figure with proportions for Church façade (Ferreira, 2015)

2.3. Geometric Patterns and Architecture

An important area in the application of geometry is that of architecture and sculpture. This has been seen from the Greeks, Romans then to Islamic architecture and so continues today.

Composition of geometric forms in architecture is a combination of space syntax (arrangement and combination of forms with each other), semiotics (the meaning and representation), and pragmatics (the effect on people). The early Greek architectures or master builders of Classical, Neo-classical even later followed by Renaissance era concentrated on human figure proportions. The admiration of Man's figure created the narcissism which became the obstacle to the human imagination. The emerge of Islam and its art and architecture, which can't be based on the human figure or image of living things, had gone beyond revealing itself in the abstract imagination ability of the mind. This understanding of geometry and geometric patterns from a different perspective is great contribution of Muslims to the world of art and architecture.

The interpretation of using geometric patterns in architecture has been given many explanations by scholars. N. Ardalan and L. Bakhtiar construed geometric patterns as

“These shapes, as the personality of numbers are understood by traditional man as aspect of the multiplicity of the Creator” (Ardalan & L., 1973, p. 40) .

As mentioned above the use of geometry is a core for architecture. The types of geometry used in architecture mainly include Euclidean and Non-Euclidean geometry, Analytic geometry, Fractal geometry, and Descriptive geometry.

2.3.1. Euclidean and Non- Euclidean geometry

Euclid, in his element formalized geometry by giving a set of five postulates initially providing the definition of the elements. ¹

On the other hand, Non-Euclidean geometry proves two parallel lines never meet at the same time showing how they can meet at a point, which the parallel postulate of Euclidian doesn’t hold. Non-Euclidean geometry is divided as Spherical geometry and Hyperbolic geometry. “Spherical Geometry is the geometry of points on the two-dimensional surface of a three-dimensional ball. Lines are defined as great circle” (Kaplan & Salesin, 2004, p. 100).

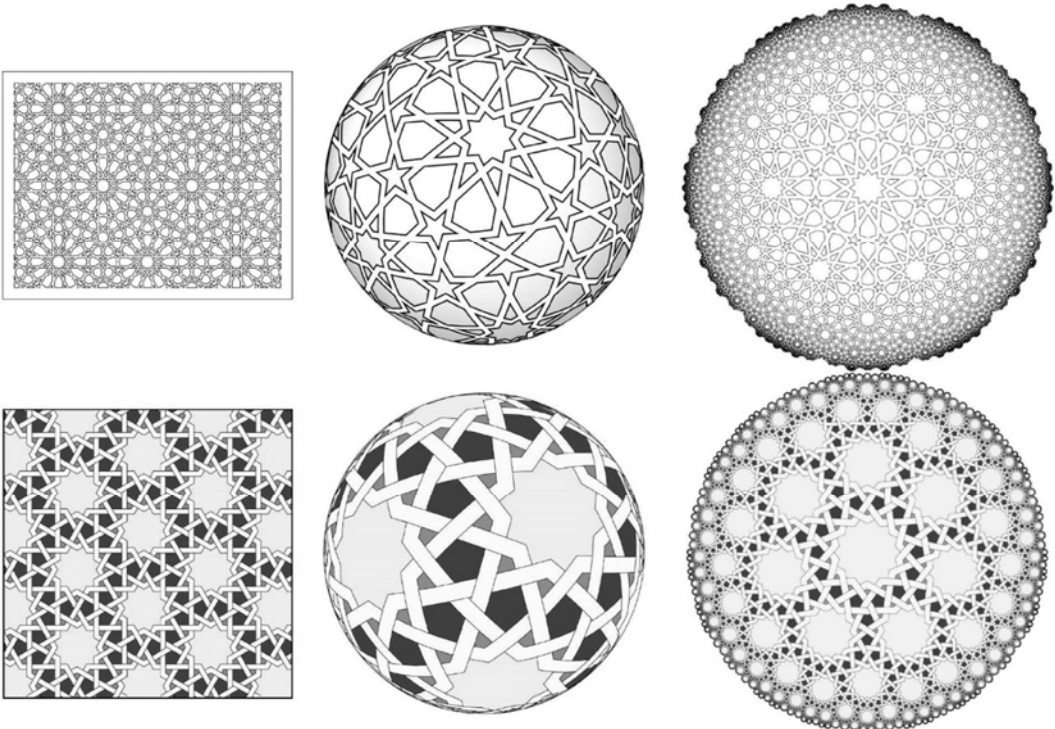


Figure 2.3. Patterns from Islamic art in Euclidean, Spherical and Hyperbolic Geometries left to right, (Kaplan & Salesin, 2004)

¹ For details of Euclidean geometry refer, Weisstein, Eric W. "Euclid's Postulates." From MathWorld--A Wolfram Web Resource. <http://mathworld.wolfram.com/EuclidsPostulates.html>

In Hyperbolic geometry the parallel postulate of Euclidean geometry is replaced with “For any given line R and point P not on R, in the plane containing both line R and point P there are at least two distinct lines through P that do not intersect R”.

These geometries are very important in studying Islamic geometric Patterns. The concepts of three-dimensional Islamic geometric patterns use Non-Euclidean geometry.

2.3.2. Analytic geometry

Analytic geometry is a classical mathematics that studies geometry using coordinates, which allows math problems in algebra to be treated geometrically and vice versa. Points are represented in plane by pairs of numbers. *Lines* and *curves* are viewed as locus of points moving according to specific equation. All Euclidean geometry can be included in Analytic geometry. In summary, Analytic geometry includes three-dimension Cartesian system, which the axes are labeled x, y, and z.

2.3.3. Fractal geometry

“Fractal geometry is the formal study of mathematical shapes that display a progression of never ending self-similar, circuitous details from large to small scale” (Yilmaz, 1999, p. 10).

B. Mandelbrot (1924-2010 CE), a Polish born French- American mathematician, proved the possibility to define all object found in nature using Fractal geometry, stating Euclidian geometry runs short to do so.

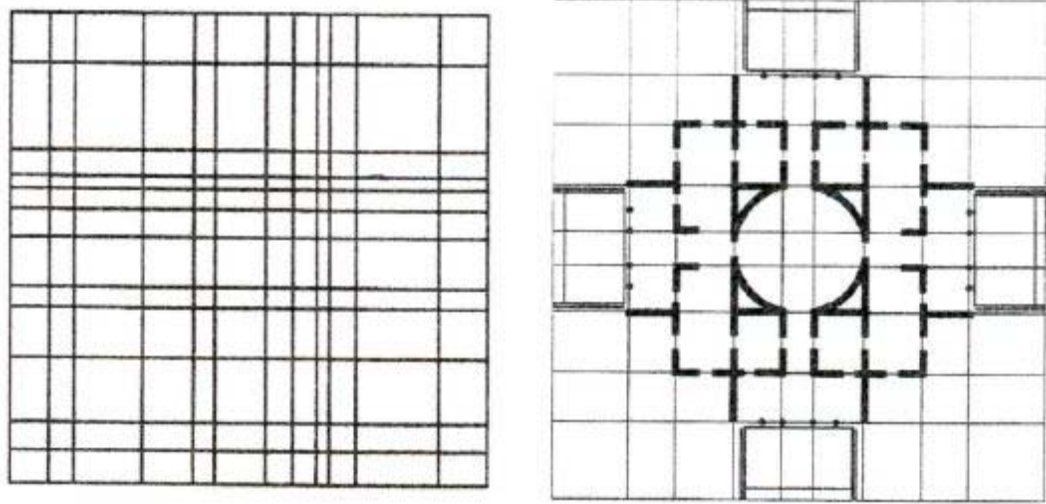


Figure 2.4. *Fractal Planning grid based on the Golden proportion fractal rhythm of Villa Rotunda, (Yilmaz, 1999)*

2.3.4. Descriptive geometry

Descriptive geometry is a geometry tool which allows to represent three-dimensional objects in to two-dimension platform using specific projection procedures. Its principles are discovered by Gaspard Monge (1746-1818 CE). It became part of the French engineering and architectural education around late 1790's.

A.P. Gomez discusses that descriptive geometry provides the knowledge of form in both decoration and stability of the building. This led to the conclusion as the architect could be able to decide the exact form and composition of his building and its parts, after understanding the descriptive geometry to the different arts and crafts.²

² For further details see: A.P. Gomez (1999), *Architecture and The Crisis of Modern Science*, London: The MIT Press.

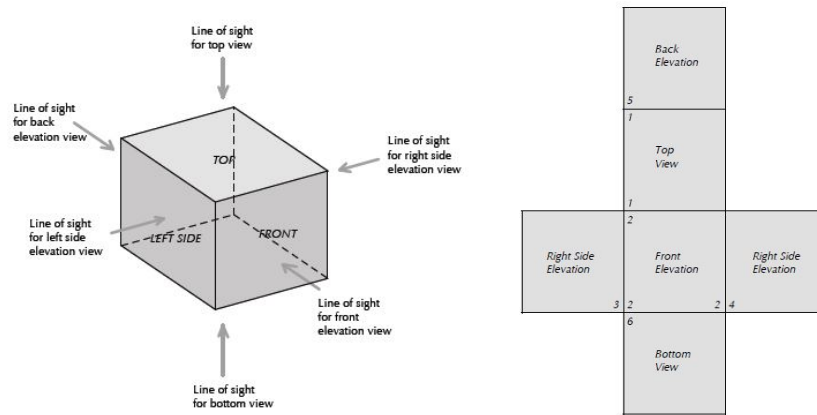


Figure 2.5. Descriptive geometry basic expression of representing the 3D object into 2D projection, (<https://www.andrew.cmu.edu/user/ramesh/teaching/course/48-175/lectures/2.BasicsOfDescriptiveGeometry.pdf>)

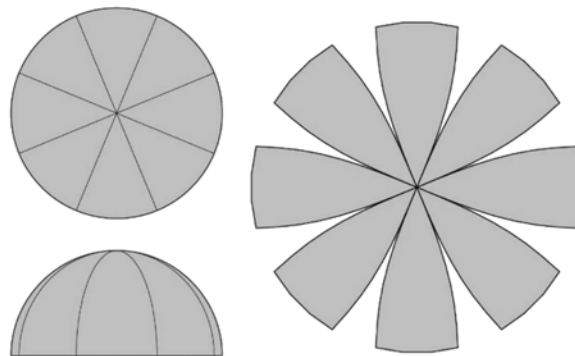


Figure 2.6. Surface of a dome represented in 2D format, (Bonner, 2018)

Previously different trend was followed combining geometry and patterns in architectures expressing the level of intelligence, priority in life and interests merging with technological advancements of eras, which is briefly discussed in this chapter. Different geometry and patterns arrangements has been seen in the surviving architectural buildings of ancient times like the great pyramids of Egypt. Nevertheless, the advancement of geometric patterns supported by mathematical calculations was advanced by Muslim scholars at the golden ages of Islam and integrated in their art works and architecture giving a diverse point of view.

In *Les Éléments de l'Art Arabe (The elements of Arab art: the trait of interlacing)* Jules Bourgoïn identified three major styles of ornamentation those are: the Greek style- corresponding to animal figures being the first. The second is, Arab style - abstract

representation with its rigid base of geometry denoted as to the crystallization of minerals. Followed by, the Japanese or Chinese style with its base of vegetable kingdoms of nature as third. Which clearly shows the importance of geometry in Islamic architecture.

3. GEOMETRIC PATTERNS IN ISLAMIC ARCHITECTURE

Geometric patterns are important characterization of Islamic architecture that makes the observer confused of whether it is complex abstract art only decorative or something beyond and sacred with more meaning, besides being mathematical and scientific. Albert Gayet, a French archeologist of Antinopolis, identifies the dominant use polygons in geometric compositions as the ‘essential character of all Islamic art’.

Titus Burckhardt (1908-1984 CE), German Swiss, who devoted all his life to the study and exposition of the different aspects of Wisdom and tradition with many publications, categorized arabesque into two. “One of them geometrical interlacing made up of a multitude of geometrical stars, the rays of which join into an intricate and endless pattern” (Necipoğlu, 1995, p. 78), which contemplates state of mind driven from *Tawhid* ‘unity in multiplicity and multiplicity in unity’. And the second one floral motif compositions not directly resembling to nature but following the laws of rhythm transported into graphic mode. (Burckhardt, Nasr, & Michon, *Art of Islam, Language and Meaning*, First ed. 1976, Commemorative ed. 2009, p. 227)

Geometry in Islamic art represents a paradigm shift in the human thought from direct representation of thing as they appear to an abstract form giving imagination enhancement and unprecedented freedom to create.³

The inspiration and significance for the use of Geometric patterns as an abstract way has been given numerous explanations by several researchers and scholars. But there is no concrete evidence for the inspiration, or mystical significance of using geometric patterns, since the artists and buildings did document or record their justifications for their principles. Regardless, by studying the basics of Islam, i.e. the Quran and Sunna, with common characteristics of Islamic art and architecture throughout, making geometric patterns their common language raised the curiosity of finding the principle. Several theories are formulated hence.

Rasheed Araeen, an Urdu conceptual artist based in London and civil engineer, suggests in his article about the significance of geometry in Islamic thoughts, Islamic patterns “were not just exercise in pattern making, to fill the empty space but rather imagination of what was invisible to the eye”. Araeen summarizes it in his words as:

³ Though in here I am not saying the ability of abstract representation of the mind is only through Geometric Islamic art or Pattern.

“Geometry in Islamic art thus represents a paradigm shift in the evolution of human thought from the observation of things and their representation as they appear to the eye to the creation of an art form whose sensuousness is the product of pure abstract thinking, thus giving the imagination enhances power to think but also unprecedented freedom to create”. (Araeen, 2010, p. 513)

On the other hand, Jules Bourgion described the geometric character of Arabs art as:

Elegance and complexity by geometric involutions more or less distinct or mixed, and constructed with symmetry. Abstract figures, linear inflections, and sort of organic growth: in other words, purely geometric themes that are graphically translated by working drawings, and technical executed by being transformed onto materials, such is the essential basis of Arab art. (Bourgion, 1879, pp. 5-11)

According to Necipoğlu Islamic geometric pattern is the subset of Islamic architecture. She wrote about the need and origin of geometrical patterns like this.

Since God and the contents of God’s eternal speech embodied in the uncreated Koran were beyond imitation, artists had no recourse but to turn to the abstract imitation of the creator realm of nature extending from the upper limits of the heavens to the earth below, a realm replete with the indirect signs and portents of God’s wisdom and infinite power. (Necipoğlu, 1995, p. 103)

She also mentions “Prisse d’ Avennes was the first to observe, in *L’art arabe*, that the usually complex Islamic geometric patterns had to be based on a system of scientific knowledge transmitted by treatises on applied geometry” (Necipoğlu, 1995, p. 68). Araeen also supports this idea and concluded as “the artistic form of geometry not only represents the ability of the mind to deal with complex problems of an abstract nature, but it demands that we look at things and understand them through the rationality of science” (Araeen, 2010, p. 518).

In Islamic geometric patterns circle is the base of all creations as symbolical representation of unity. The expansion and regular division into equilateral polygons, is the starting base of Islamic geometric patterns. When the corners of these constructive polygons are connected star-polygons are formed and from that more complex structure of rosette. This geometric based art is what is referred as ‘Islamic Geometric Pattern’.

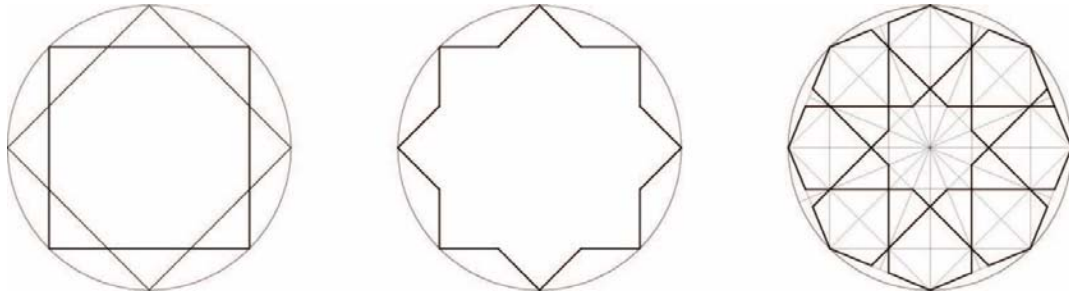


Figure 3.1. *The development from regular polygons inscribed in the circle to star and rosette*

The development of Islamic geometric patterns started with such a simple form of polygons inside a circle enhanced to one of the most complex structures through time and deeper knowledge of geometry and math, especially after the translation of number of Greek books to Arabic in different fields of knowledge at the ninth and tenth century in Bagdad.⁴ From the 7th to 9th century significant original contributions have not been made by Muslims, but this doesn't mean that there are no progresses. By the 8th and 9th century Muslims had grown scientific and mathematical knowledge besides technology is the base for expansion and complexity of geometry in Islamic art and architecture. Muslims did not just copy what they got from the Greek's or Roman's study of Geometry and math they advanced it with understand in depth and applied it contributing to modern math, geometry, art and architecture moreover than other fields.

Islamic geometric pattern is characterized by networks of interlocking stars and polygons. Even though the fundamental way of construction is not complex, it elaborates a form of geometric art providing enormously rich source of types restricted only to the imagination and knowledge of the designer. While there are many complex pieces of Islamic geometric patterns, there are no text descriptions that explain the design principle of these Islamic art and architecture.

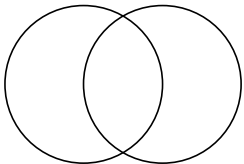
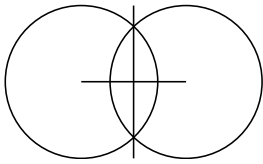
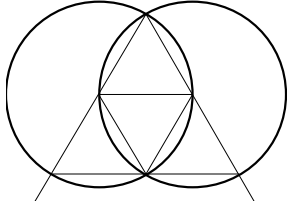
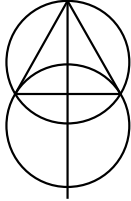
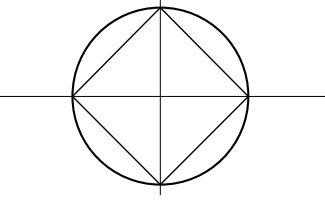
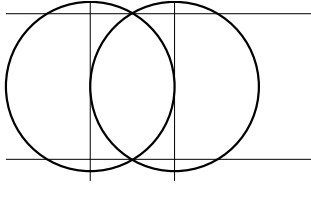
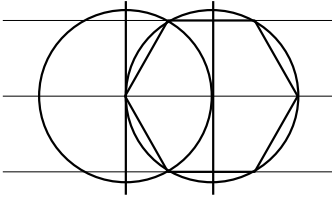
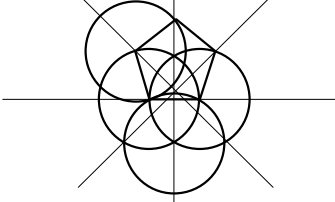
As from the many classifications of Islamic geometric patterns, based on appearance in this research paper it has being grouped as two-dimensional patters and three-dimensional patterns or forms.

⁴ For the details of how this has affected the geometrical use in Islamic art and architecture see chapter 8 in: Necipoğlu, *op. cit.* 1995, pp. 131-166

3.1. Two -Dimensional Islamic Geometric Patterns

The two-dimensional pattern includes all the Islamic geometrical patterns that are designed and applied in the decoration, carvings and rendering of surfaces in general. Not only limited to this but also the bases of three-dimensional patterns are also 2D patterns. Islamic geometric patterns are formed by using pair of compasses and a ruler, starting with a circle that represents unity. Circle has a great significance in drawing different types of patterns.

Table 3.1. *Significance of Circle, (Reki & Selçuk, 2018)*

Vesica piscis		
The in-triangle		
The in-square and rectangle		
The hexagon and pentagon		

According to (El-Said & Parman, 1976) all Islamic arts and architectural forms, especially geometric pattern design, are based on the proportions of the golden mean,

primary three proportional roots i.e. $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$. For detail mathematical understanding of the proportional roots refer (Lawlor, 1982, pp. 23-52).

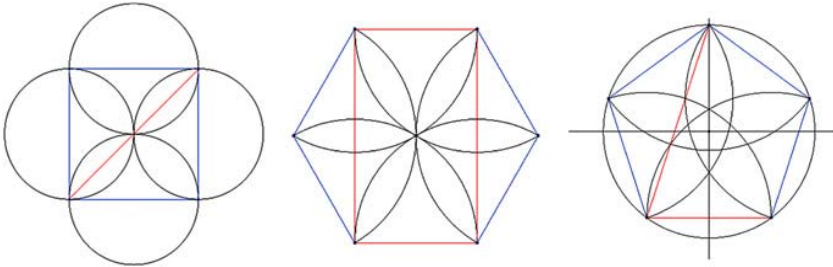


Figure 3.2. *The proportional roots: (a) the $\sqrt{2}$ proportion, (b) the $\sqrt{3}$ proportion, and (c) the golden mean (Phi) proportion. (El-Said & Parman, 1976)*

3.1.1. Division of Two-dimensional Islamic geometric Patterns

Two-dimensional IGP are classified based on the ratio relations of the diagonals. They are mainly divided as $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ proportions as many scholars has mentioned.

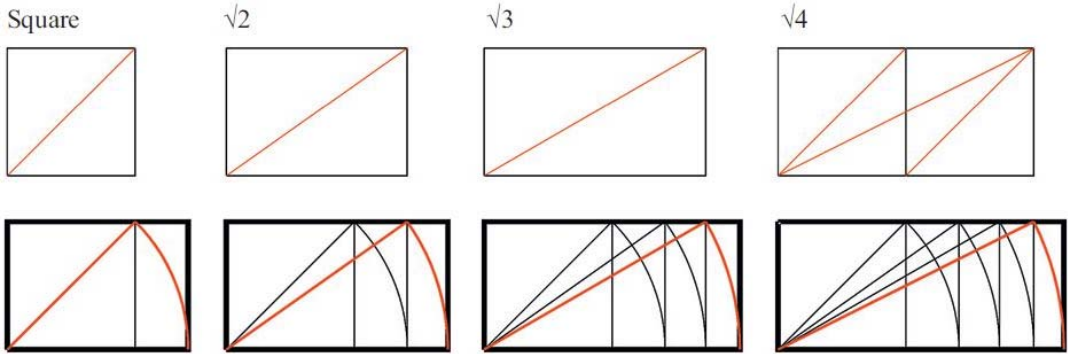


Figure 3.3. *The root proportions based on the square. (Dabbour, 2012)*

3.1.1.1. The class of $\sqrt{2}$ relations

The tiling of these patterns is based on square plane. Most common examples of this class are octagon-based or eight-point geometric patterns.

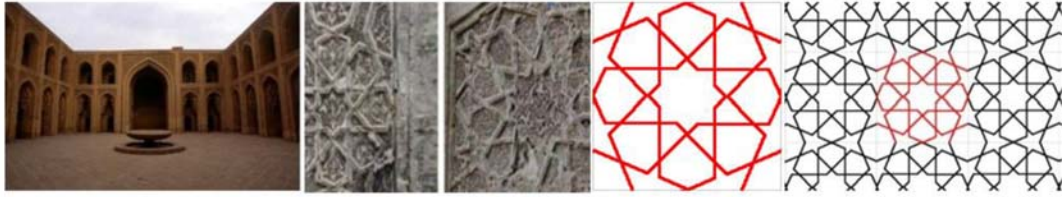


Figure 3.4. (Left) Abbasid Palace, courtyard in the House of Wisdom, known as the Bait al-Hikma. The second image is the original decorations before restoration. The basic 8-fold rosette pattern, found in the ornamentation. (right), (Reki and Selçuk, 2018)

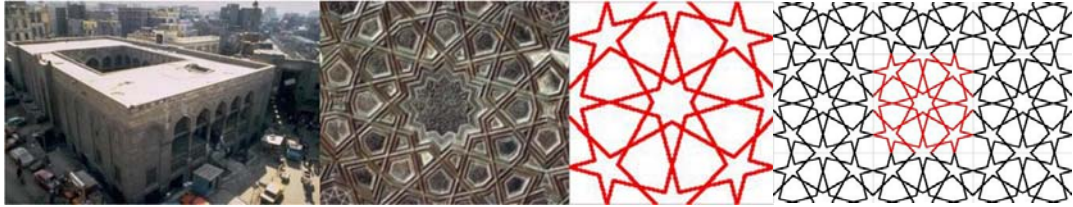


Figure 3.5. Mosque of Al-Salih Tala'i the fold rosette pattern detail in the mesjid's interior. (Reki and Selçuk, 2018)



Figure 3.6. 16 point-based Rosette patterns widely used in Morocco (https://www.essaouira.nu/art_symbols.html)

Complex forms that evolved later as 16-point geometric patterns are also results of root two relations.

3.1.1.2. The class of $\sqrt{3}$ relations

The tiling of these patterns is based on hexagonal plane (Figure 3.7). Most common examples of this class are six or twelve-point geometric patterns.

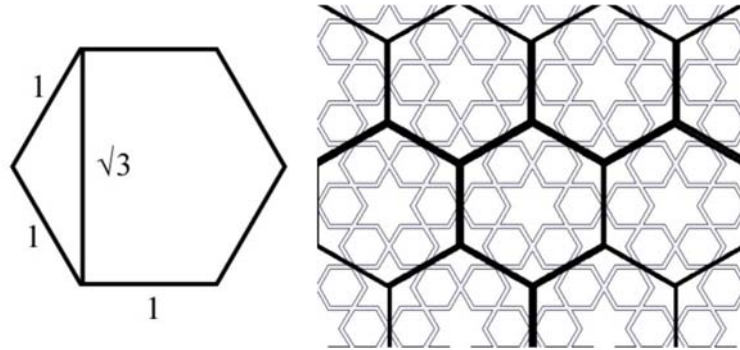


Figure 3.7. (Left) Proportion in a hexagon, (Right) A pattern hexagon regular tiling

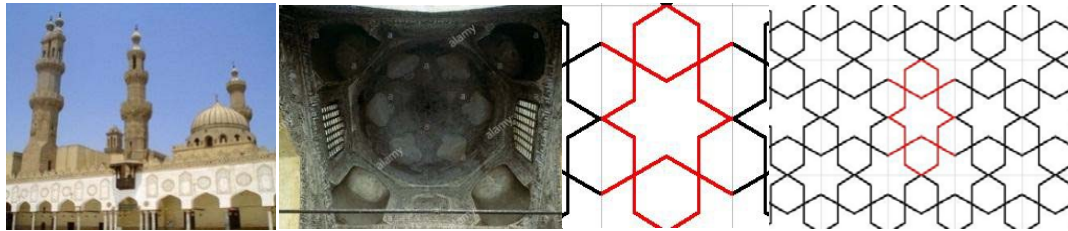


Figure 3.8. Al-Azhar Mosque and the detail of the dome showing a six-point geometrical pattern, (Reki & Selçuk, 2018)

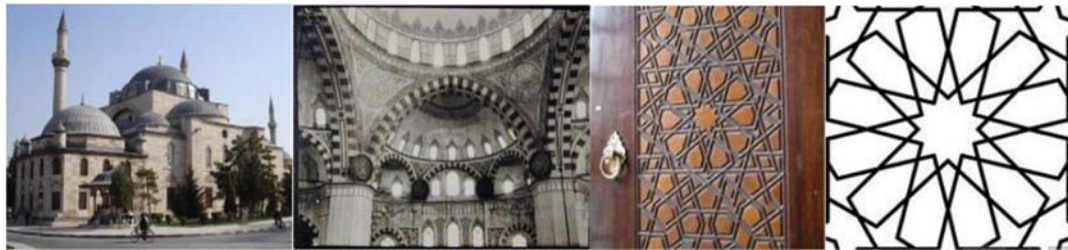


Figure 3.9. Sehzade Complex showing twelve-point geometric pattern, Istanbul, (Reki & Selçuk, 2018)

3.1.1.3. The class of $\sqrt{5}$ relations

The patterns of $\sqrt{5}$ relations are ten pointed geometric patterns which are based on pentagons. However, since pentagons can't make tessellations by themselves and fill the

plane regularly, they were either mixed with other patterns or used as a center piece to fill smaller planes. The pattern of $\sqrt{5}$ relation is the same value as golden proportion.



Figure 3.10. (Left) The ratio of pentagon, which is golden proportion, (Right) ten-pointed geometric pattern from root five relations

But what should be noted is not all types of Islamic geometric patterns can be based on primary three proportional roots. There are designs with 9-, 11-, 13- and 18- pointed stars and geometric patterns. These types of patterns can't make regular tessellation of one type. P.R. Cromwell in his mathematical analysis of the Topkapi Scroll describes it as: "this preference for generating designs by reflecting rectangular templates is another factor that precludes the production of 3-fold symmetry" (Cromwell, 2010, p. 75).

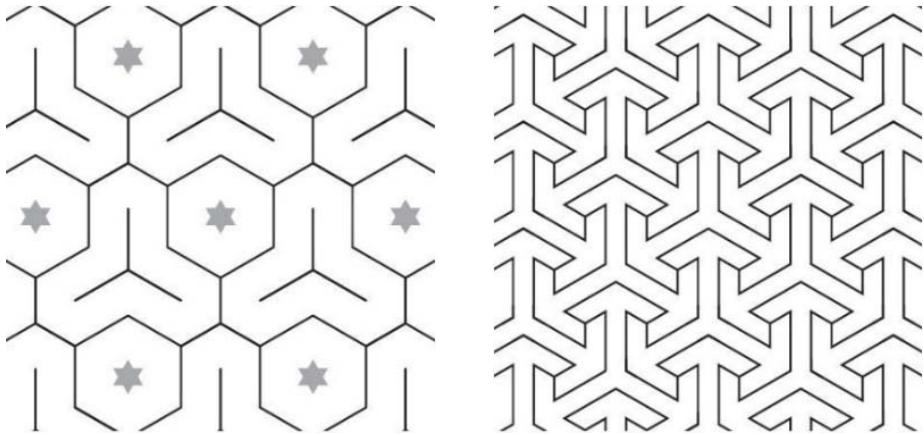


Figure 3.11. Some 2D Islamic Patterns with 3-fold symmetry, (Cromwell, 2010)

3.1.2. Evolution of Two- dimensional Islamic Geometric Patterns

Islamic geometric patterns have evolved in advancement and new innovations for up to the 14th century and in the western side of the Islamic world up to beginning of 16th century as both Necipoğlu and Yaghan explained. Regarding the art of geometric patterns in the Muslim world currently; Araeen indicates that “the Muslim world has now been emptied of its spirits and turned into the decorative patterns ... representing the intellectual vacuousness of the Muslim world in general today” (Araeen, 2010, p. 517).

The evolution of 2D IGP when analyzed through Islamic Dynasties, that had major contribution in shaping the Islamic geometric pattern, with their capitals is as follows:

- Abbasid: 750-1258 CE (Golden age of Islam)
 - Anbar Kufa: (750-762)
 - Baghdad: (762-796 CE)
 - Raqqa: (769-809 CE)
 - Baghdad: (809-836 CE)
 - Samarra: (836-892 CE)
 - Baghdad: (892-1258 CE)
- Seljuk Empire: (1038-1194 CE)
 - Nishapur: (1037-1043 CE)
 - Rey: (1043-1051 CE)
 - Isfahan: (1051-1118 CE)
 - Merv (Eastern Capital): (1118-1153 CE)
 - Hamadan (Western Capital): (1118-1194 CE)
- Mamluks: Cairo 1250-1517 CE (Eygpt & Syria)

Umayyad (661-750 CE) was the first great Muslim dynasty to rule the empire of the caliphate basing in the current day Syria. The art of decoration was based on vegetal and flora patterns influenced by inspirations from Byzantine and Sassanian empires technique. According to the survey of Y. Abdullahi and M.R.B. Embi there is no sign of the use of geometrical motifs. (Abdullahi & Embi, 2013, p. 245)

Great Mosque of Kairouan in Tunisia is a good example of Abbasid buildings, with primarily flora motif ornamentations combined with elementary geometrical shapes. Earliest example for woven geometrical patterns in Muslim ornamentation are simple 6- and 8-point geometrical patterns used in the Mosque of Ibn-Tulun. End of the 9th and

beginning of 10th century, geometric motifs become extensively influential in art and architecture leading to the use of 3D geometric patterns as well. The Abbasid Palace in Baghdad and the Madrasa of Mustansiriyah are examples of this decorated with Muqarnas.⁵



Figure 3.12. *The Abbasid Palace in Baghdad (1230CE), (photo: from the collection of Eric Strating, DHM of the Netherlands in the Embassy to Iraq)*

The Seljuk's transformed geometric motifs Islamic art and architecture by introducing sophisticated complex patterns. Six and eight-point geometrical patterns, use of ten-point pattern and patterns made of non-constructible polygons as 7, 9, 11 and 13-point geometric patterns are all observed in the Seljuk's architecture. From the early Seljuk's architecture, tomb towers of Kharraqan (1067-1093 CE) in the regions of Northern Iran, are examples to mention. "It has an octagonal plan, with double crusted domes and brick walls along highly ornamented panels with different abstract and star geometrical patterns on the façade" (Reki & Selçuk, 2018, p. 88). The Friday mosque of

⁵ The Abbasid Palace in Baghdad, believed to have been built by Calipha Al-Nasser, in the 1230 CE. And the Madrasa of Mustansiriyah, built around 1233 CE, is one of the only buildings still standing, being the evidence of Abbasids contribution to the Islamic geometric patterns in Islamic architecture.

Isfahan is also excellent example of Seljuk style, incorporating mostly of 5 and 8-point star patterns.



Figure 3.13. *The tomb towers of Kharraqan (1067-1093 CE) (Photo: from Wikipedia)*

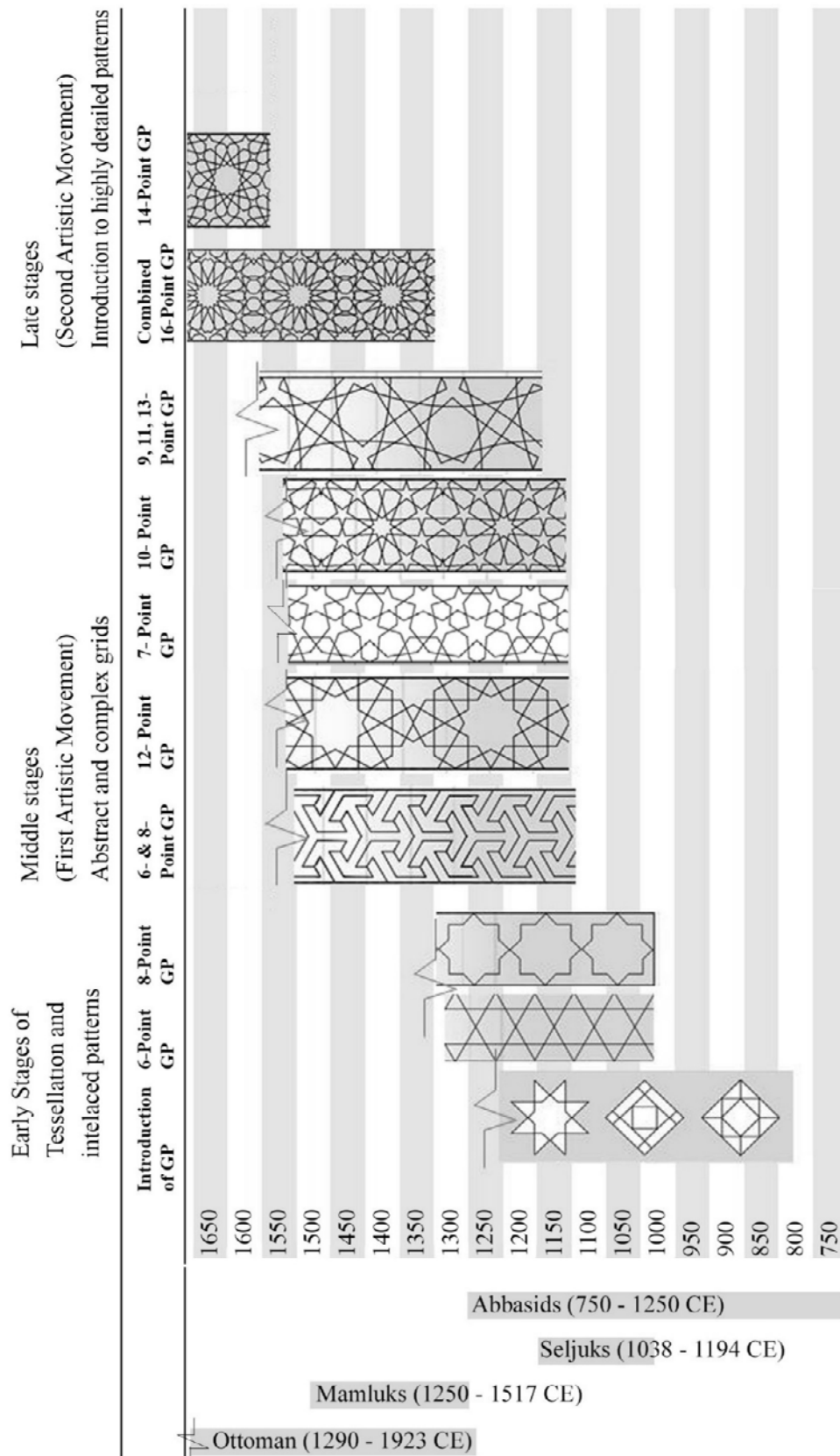


Figure 3.14. Time chart of the evolution of IGP and their co-existence in certain eras.

3.2. Three-Dimensional Islamic Geometric Patterns

Three-dimensional geometrical patterns are the transformations of the two-dimension geometric patterns with the addition of plane into forms or object pieces standing by itself either as part of the architecture or directly being the structure by itself. These patterns are defined by the two-dimensional patterns plans. 2DPP are defined by Yaghan in his thesis as “a two-dimensional geometrical pattern that divides an area clearly defined by certain boundaries (which are the projections of muqarnas boundaries) into many connected compartments without leaving any gaps in between” (Yaghan, 2001, p. 17).

The tessellation of the patterns also follows a certain order of rules with a single or multiple focal point. The compartments are also pure or distorted geometric shapes. Three-dimensional patterns are also what is referred as muqarnas.

These patterns are applied in Non-Euclidean geometry with also further complexity of a 3D form attachments to the sphere or hyperbolic geometries in some cases. The origin, progress and evolution are discussed in detail below.

3.2.1. Origin of 3D Islamic Geometric Patterns

The starting period of three-dimensional IGP as identified by many scholars is 9th and 10th century. Necipoğlu argues, while analyzing a 12th century text of Ibn Jubayr that “Ibn Jubayr’s description suggests that intricately joined two-dimensional star and polygon patterns may originally have been considered an integral part of *qarbasa* or *qarnasa* work before muqarnas become a more specialized term referring to ‘*talacties*’” (Necipoğlu, 1995, p. 173). Also, L. Golombek and D.N. Wiber state that “in Arabic literature, the term Muqarnas described a variety of different techniques; painting, carving, or very elaborate decoration” (Golombek & Wilber, 1988, p. 106) though no examples are supplied.

M. Ecochard (1905 -1985 CE), a French urban planner and Architect, who played an important role in the restoration and redevelopment of Syria, Morocco, Lebanon and others, introduced squinches in the 10th century as the functional origin of muqarnas,

which is the term used widely after 11th century to describe the complex 3D Islamic geometric pattern.⁶ However didn't not explain the evolution and gradual change.

Three-dimensional Islamic geometric patterns were primary applied as structural or decorative method of geometrical experimentation. These descriptions include patterns forms (Muqarnas) in dome-transitional zone (corner arch, squinch, and pendentive), cornice, balcony supports corbels and ribbed domes and vaults that are self-standing or attached to surfaces.

There are three issues to distinguished according to (Yaghan, 2001) while studying the origin of three-dimensional IGP, specially muqarnas, i.e.: Chronological, geographical and formal origins. The chronological and geographical origins of early three-dimensional pattern types are determined through architectural monuments and archeological findings. Formal origin means the form or forms which muqarnas are derived or inspired by.

3.2.1.1. Chronological origin

Chronologically origin of forms from archeological finding earliest samples of muqarnas date to the 9th and 10th century. By late 11th and early 12th century they were practiced all over the Islamic lands. This shows, that muqarnas works come after a complex works of two-dimension Islamic geometric patterns advancing to three-dimensional.

3.2.1.2. Geographical origin

As for the geographical origin there are two theories. One is it has originated from a single place and spread through. The second theory is, 3D Islamic geometric pattern has developed in multiple regions simultaneously. Scholars who supports the idea of single geographical origin suggests two places as origins: Eastern Iran and Baghdad.

Based on the archeological findings at Nishapur and Tim (the figure below), early examples of muqarnas, Eastern Iran is suggested as the geographical origin. Scholar as D. Behrens-Abouseif and G. Necipoğlu support the single origin theory.

Baghdad, being the city where the translation of many books from Greek to Arabic took place, is considered since it is the capital and cultural center of Islamic world

⁶ M. Ecochard (1977), *Filiation de monuments grecs, byzantins et islamiques: une question de géométrie*. Paris: Paul Geuthner

therefore, new innovations should also come from there. And, the findings of mausoleum of Imam Dur and Fustat, existing domes totally built of muqarnas units, in Baghdad from the Abbasids period supports this theory.

Nevertheless, the theory of multiple origins simultaneously has more logical grounds. O. Grabar in his book 'The Alhambra' suggests muqarnas was created in an uninformed simultaneous development in eastern Iran and central North Africa. (Grabar O. , 1978, p. 176)



Figure 3.15. Arab-Ata mausoleum (977 CE), Tim Uzbekistan.

3.2.1.3. Formal origin

Depending on the purpose of creation it has been difficult to determine three-dimensional Islamic geometric patterns as decorative or structural. Its purpose started as structural and later on became decorative as geometric experimentation, making the previous works of 3D Islamic geometric patterns in the 10th and 11th century simpler, less decorative and more structural. Patterns that evolved from domes as structural load transferring components are to be considered the older ones. Many forms appear to have inspired the creation depending upon the place of use. Yaghan has divided the formal origin from its role of creation as: Dome-transitional zone forms, Ribbed vaults, Cornice, balcony support and corbels.

Dome-transitional zone forms developed from dome, that is the fitting of a circular structure on top of a rectangular or square form.

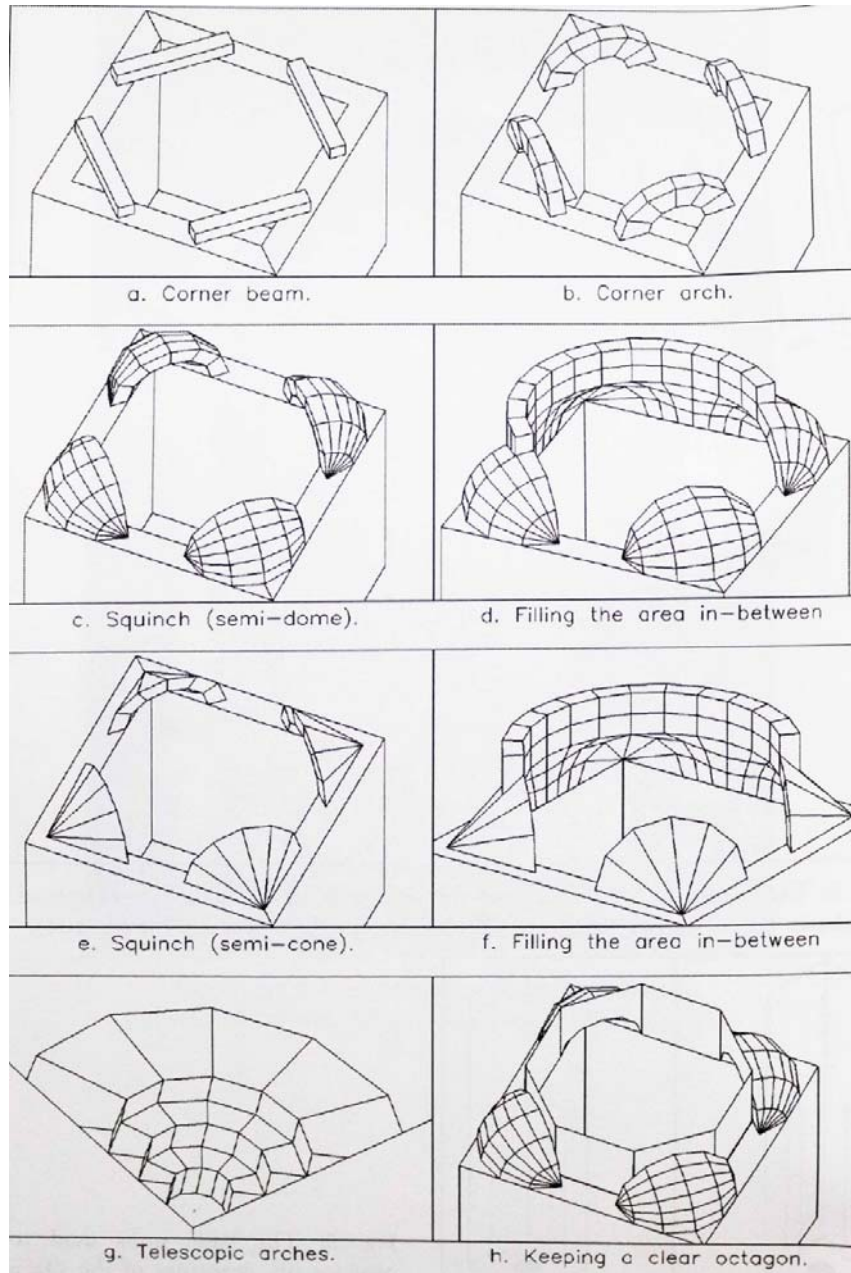


Figure 3.16. *Dome-transitional zone of formal origin of Muqarnas (M.A. Yaghan, 2001)*

3.2.2. Evolution of Three-Dimensional Islamic geometric patterns

The origin of three-dimensional Islamic geometric patterns has been discussed previously continuing with the progress and evolution is represented in the diagram below for this thesis based on the argument of its origin being evolvement from combination of two-dimensional patterns and forms in different geographies. The fact that the term Muqarnas was first used at the beginning of the 11th century shows that it has evolved after

the advancement of geometry and other science fields knowledge in the Islamic world. After the 16th century there are no major innovations in the evolvement of muqarnas according to M.A. Yaghan⁷. Necipoğlu also mentions although efforts have been made to add innovative designs, one can conclude that since the 14th century, Moroccan Muqarnas have not changed and creativity can only be observed novel proportion of elements and tiers in the structure.⁸

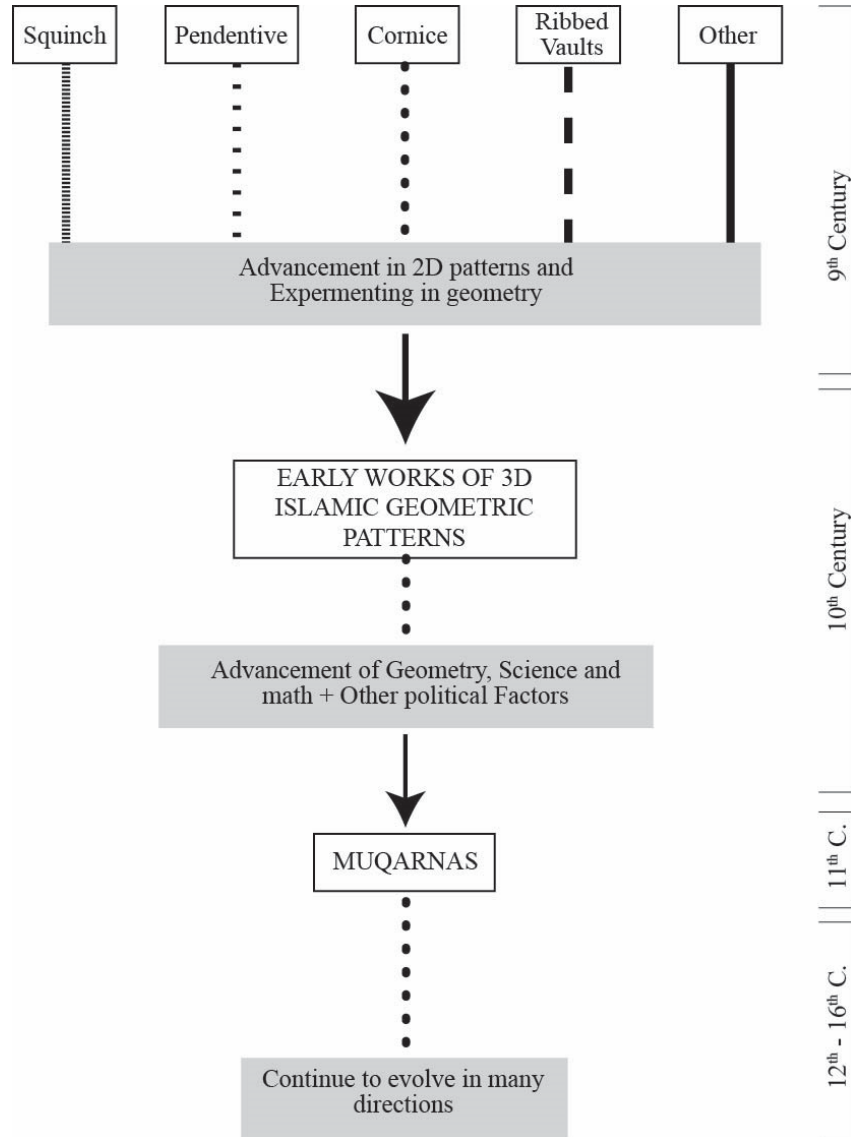


Figure 3.17. Graphical representation of the origin of three-dimensional Islamic geometric patterns and primary evolvements

⁷ For the detail see: M.A. Yaghan (2010), The Evolution of architectural forms through Computer Visualization: Muqarnas example. In *Electronic Visualization and the Arts*. London, 113-120

⁸ For the detail see: Necipoğlu, *op. cit.* 1995, pp. 125-175

3.3. Muqarnas

Muqarnas are complex three-dimensional Islamic geometric patterns, which are also used in non-Islamic buildings, like Cathedrals and palaces, due to the attraction of their structural and ornamental sophistication. Muqarnas is a “pure Islamic form” as also claimed by Yaghan (2001), the greater the understanding of muqarnas is the greater the understanding of Islamic architecture in general and use of geometry especially.

M.A. Yaghan mentions three major shortcomings in the study of muqarnas. The first is “there are no clear consistent definitions of the different issues of muqarnas, not there is a consistent terminology” (Yaghan, 2001, p. 8). Second and Third are, generalization of some contextual studies and lack of detailed research works accordingly. Necipoğlu also supports this shortcoming by mentioning that the first studies made by European scholars on Islamic art and architecture was selective and ignored three-dimensional geometric patterns as muqarnas since it didn't fit in their classification.

The term Muqarnas was not used until 11th century. Early definitions of it was given by Al-Kashi, 15th century mathematician, in his Arabic treatise ‘*Miftah al-Hisab*’ (‘Key to Arithmetic’) translated to English by Dold-Samplonius, (1992) as:

“The Muqarnas is a roofed *Musaqqaf* (vault) like a staircase *Madraj* with facets *dil*’ and a flat roof *sath*. Every facet intersects the adjacent one at either a right angle, or half a right angle, or their sum, or other than these two. The two facets can be thought of as standing on a plane parallel to the horizon. Above them is built either a flat surface, not parallel to the horizon, or two surfaces, either flat or curved, that constitute their roof. Both facets together with their roof are called one cell *bayt*. Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier *tabaqa*” (Dold-Samplonius, 1992, p. 226)

However, Yaghan while analyzing Al-Kashi's definition remarks three shortcomings:

1. The use of the term ‘roof’ or ‘roofed vault’ excludes Muqarnas forms that are applied elsewhere like column or furniture.
2. Units (cells) described are confined to certain type that has an angular base, thus the units with their base being an arc (as those used in the portal of Madrasah Al-Firdaws in Aleppo) are excluded.
3. It overlooks the role the two-dimensional patterns, according to which units are determined in the first place.⁹

⁹ Yaghan, *op. cit.* 2001, p. 13-14

Scholars like D. Behrens-Abouseif defined muqarnas overlooking the layers and structural qualities as a three-dimensional form of decoration. “Muqarnas decoration is composed of a series of niches embedded within an architectural frame, geometrically connected and forming a three-dimensional composition around a few basic axes of symmetry” (Yaghan, 2001, p. 14)¹⁰.

After many analyses of neither overlooking or excluding the different patterns, nor including other types of Islamic geometric patterns Yaghan, defines muqarnas

“A three-dimensional form, whose visual function is the gradual transition between two levels, two sizes and or two shapes. It consists of orderly horizontal Tiers of small units Unit-surface that are stacked one on top of another, connected via their bases and tops Layer-lines. The base Layer-lines of a tier and the top layer lines of the tier below are either identical or separated by Layer-joints. Any horizontal gaps in between layer-lines of same height (like stars) are filled by flat, or slightly round Roof-patches” (Yaghan, 2001, p. 18).

In the studying of muqarnas by O. Graber in his book *Alhambra*, mentions its character of being a 3D provides nature and depth of volume to be determined by the maker makes it usable both as an architectonic form due to its relationship to vault and as ornamentation because its depth controllable. (Yaghan, 2001, p. 27)

The use of muqarnas has become universal in Islamic architecture, by late 11th and 12th century, being applied in major buildings, walls, niches, facades, arches, friezes, and entrance portals, like Mesjids, mausoleums, palaces and so on.

Depending upon their application muqarnas can be classified as structural and decorative. As mentioned above they are used in transitional zones. There types patterns include; Muqarnas in dome transition as corner arch, squinch and pendentive or cornice (Figure 3.16), balcony supports (Figure 3.19), corbels as self-standing or attached to surface.

3.3.1. Structural Muqarnas

Structural muqarnas are recognized by the characters as plans with clear distinction between the envelope and its transitional zone with single focal points. Mausoleum of Imam Dur and the tomb of Zumurrud Khatun are the selected two most known examples of their type. These two examples are selected since one is the earliest model and another is highly matured type.

¹⁰ D. Behrens-Abouseif, (1993), *Encyclopedia of Islam*. New ed., vol V.I, by E. J. Brill in: Yaghan *op. cit.* 2001, p. 14

Table 3.2. *Structural Muqarnas' of Mausoleum of Imam Dur and the tomb of Zumurrud Khatun, (The plan and section drawings were produced by Herzfeld)*



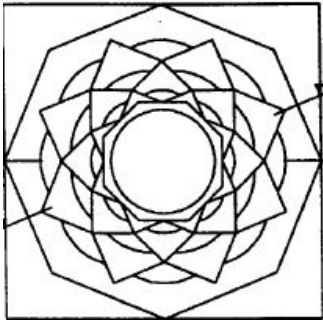
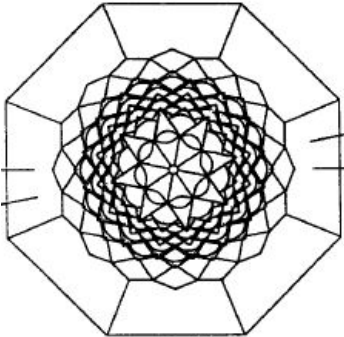
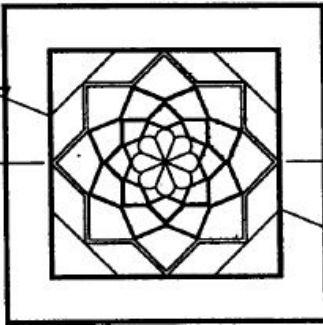
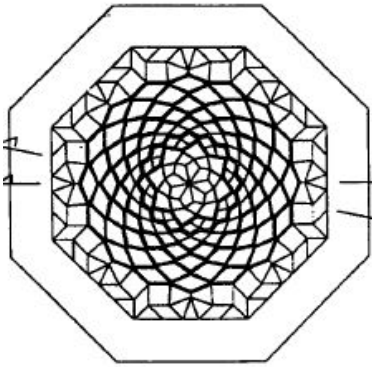
	Mausoleum of Imam Dur	Tomb of Zumurrud Khatun
Location	Iraq, Tigris	Iraq, western Baghdad
Date	(1085-1090 CE)	(1202 CE)
External look		
Roof Plan		
Plan looking up		

Table 3.3. (Continued) Structural Muqarnas' of Mausoleum of Imam Dur and the tomb of Zumurrud Khatun, (The plan and section drawings were produced by Herzfeld)

Picture looking up		
Section		
Description	<ul style="list-style-type: none"> - Radical and star composition - Single shell dome composed of muqarnas units - Echoes its interior in the exterior - Intricate decorative cladding - Has 5 tiers 	<ul style="list-style-type: none"> - By this time, maturity of form reached its peak - Orthogonal plans - Single shell dome composed of muqarnas units - Echoes its interior in the exterior - Composed of 10 tiers that are structural holding the dome and additional 3 below work as a buffer between the dome and octagonal base.

As can be seen from the two examples structural muqarnas have simpler arrangements.

3.3.2. Decorative Muqarnas

As explained above decorative muqarnas patterns generated afterward with the modifications and additions of elements in the structural ones. But this doesn't mean that structural muqarnas were not decorative as well.

Three-dimensional Islamic geometric patterns that are mentioned as examples in this paper are of decorative. Especially the ones that are built after the 12th century, that are complex, are all decorative with some exceptions used in support of balconies and under dome transitional forms. Decorative muqarnas are ornamental deadload in a building.



Figure 3.18. From Left to Right; Chamfered corner muqarnas of the Aqmar Mesjid (1125 CE) in Cairo (photo source: Wikipedia) , Masjid Jami (1105-1118 CE) or Friday Mesjid of Gulpayigan's squinch muqarnas (https://archnet.org/sites/3872/media_contents/41008), Matbakh (kitchen) al Ajami (13th c.) of Aleppo's pendentive muqarnas (https://archnet.org/sites/10559/media_contents/111592)



Figure 3.19. Left - Tomb tower of Gunbad-i 'Ali's cornice muqarnas (1056 CE) located Iran (https://archnet.org/sites/1594/media_contents/975); Right - Amir Khayrbak Funerary Complex (1502CE) minaret balcony supports corbels, (http://archnet.org/media_contents/46235)

The two classifications above are the results of advancement of the Islamic world in better understanding of math and geometry with application of it in art and architecture. the 2D IGP has evolved from simpler patterns into complex illusion figures, a similar development is shown in muqarnas as well up to 14th century as Necipoğlu proclaims and beginning of 16th century according to Yaghan.

3.3.3. Evolution of Muqarnas

Muqarnas as originating from squinches or filling form evolved from being load transferring elements to a complex decorative that has amazed the world being copied and used in cathedrals and temples as well.

The evolution of muqarnas patterns has taken place in many features, as layer adding, single unit complexity, arrangement of the units, the design curves and other elaborations. In terms of the arrangement system of patterns muqarnas forms have also evolved from simpler linear, radical and grid arrangements to complex forms.

3.3.3.1. System arrangement of Muqarnas

The arrangement of muqarnas forms or units are classified by scholars as simple linear to radical and complex. Based on the arrangement of the repeating units Yaghan divides it into four as linear, radical, grid arrangements and complex forms. (Yaghan, Muqarnas, 2001)

3.3.3.1.1. Linear (Straight or Curved) system arrangement

Linear systems are famously used in the forms adorning *minarets* but are also found elsewhere. The muqarnas at the entrance of Masjid Jami (1038-1118 CE) the Friday Mosque of Isfahan, the exact construction date is hard to determine since it has been renewed and renovated many times, is one of the top examples. It has simple linearly arranged muqarnas units over a curve; arrayed to the niche as shown below.

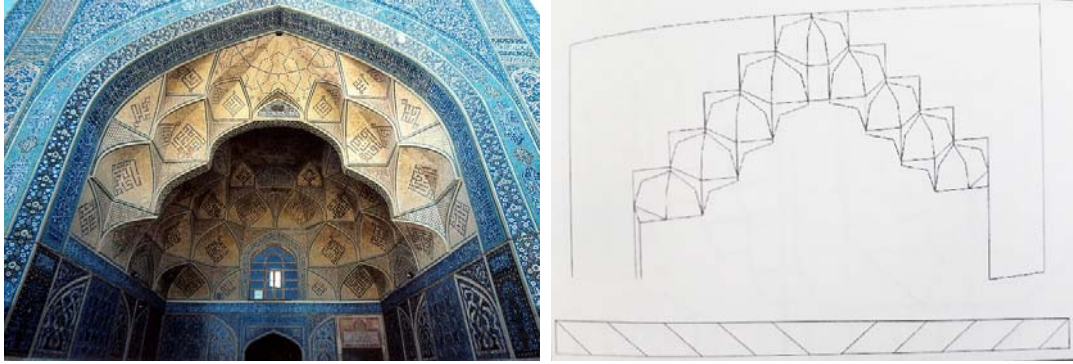


Figure 3.20. *The Muqarnas of niche in Masjid-i Jami, Linear arrangement , Left- (<https://www.khanacademy.org/humanities/ap-art-history/west-and-central-asia/a/the-great-mosque-or-masjid-e-jameh-of-isfahan>); Right – (Yaghan, 2001)*

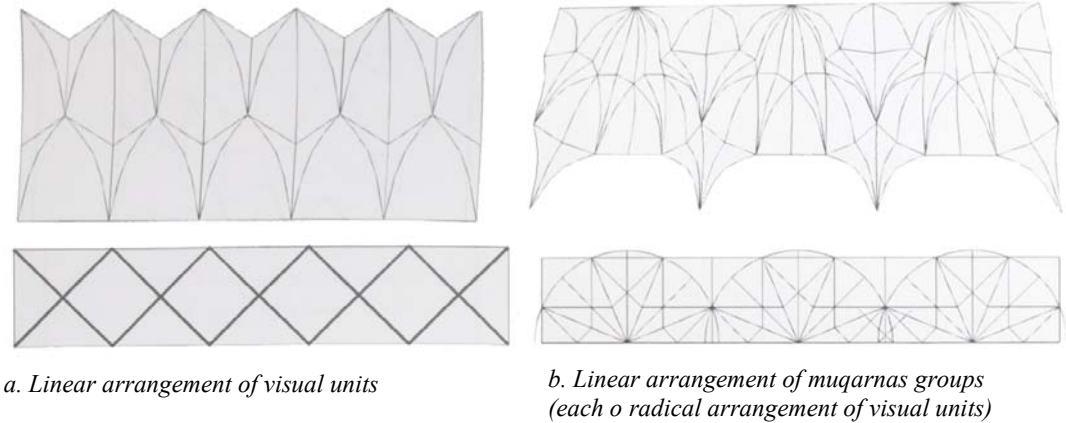
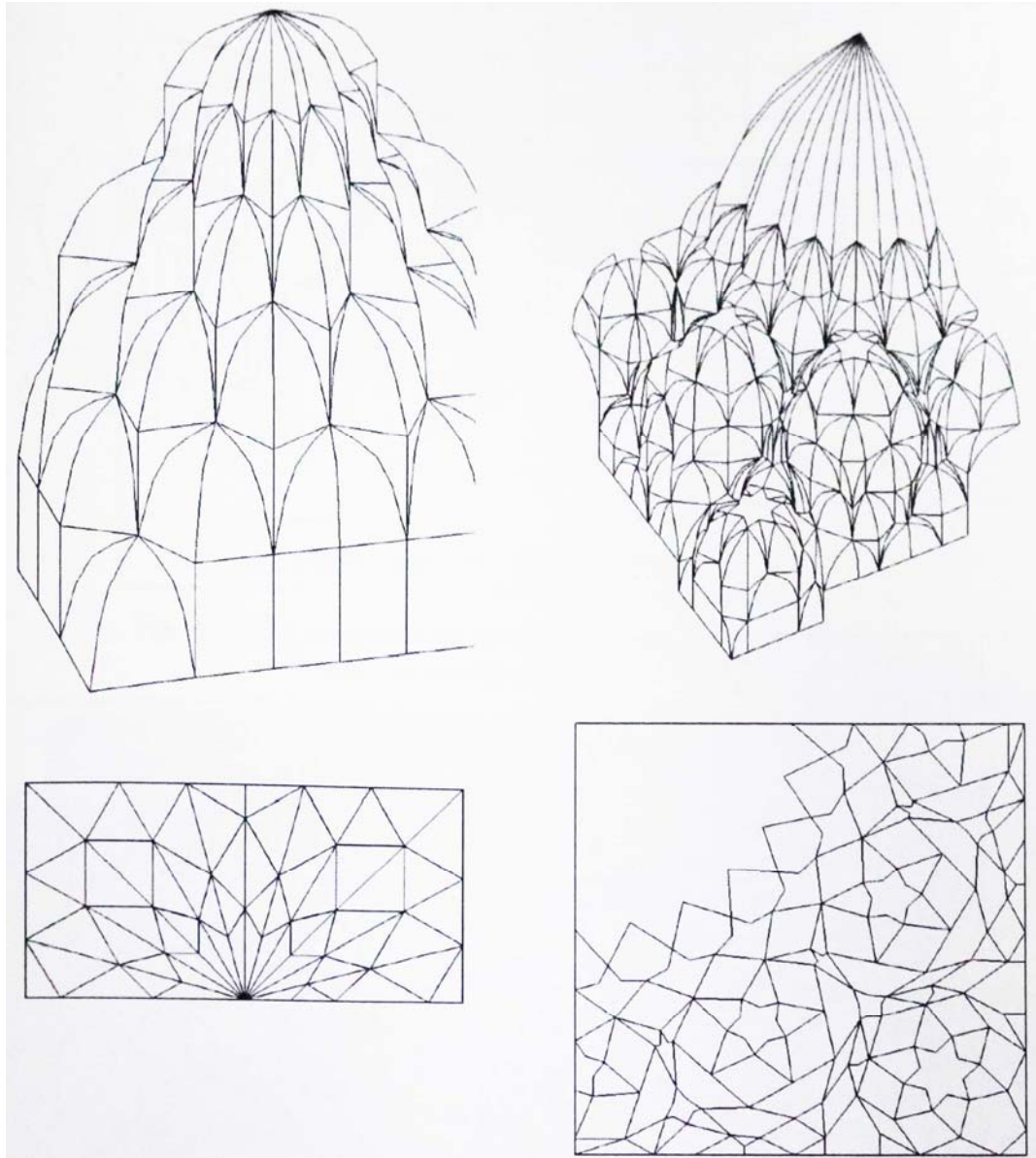


Figure 3.21. *Linear Arrangement system (Yaghan, 2001)*

3.3.3.1.2. Radical system arrangement

The Radical system of arrangement of units are mostly observed concentric stars with single focal points. Some examples can be the mausoleum of Imam Dur (Table 3.1) and muqarnases forms.



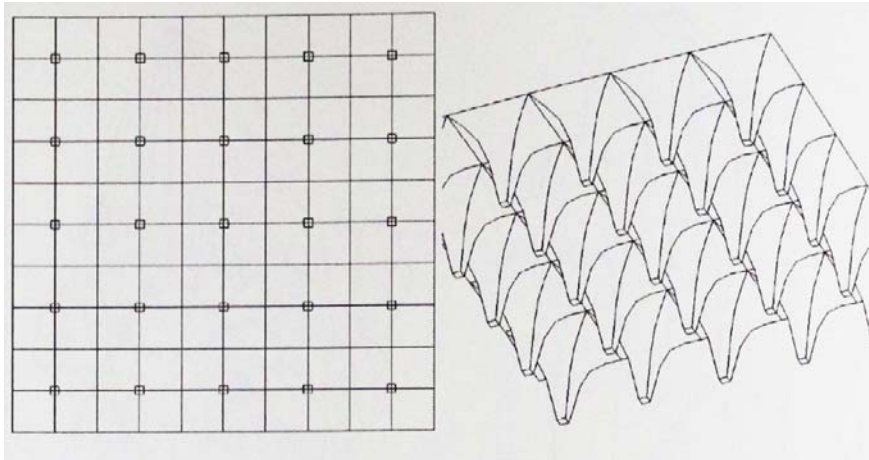
a. Radical arrangement of visual units

b. Radical arrangement of visual units and muqarnas groups (each id of radical arrangement)

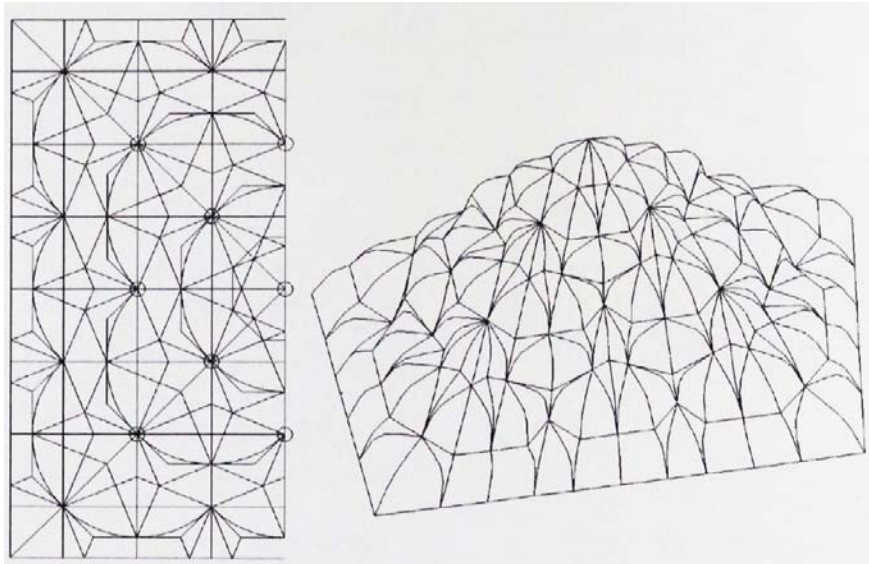
Figure 3.22. Radical Arrangement (Yaghan, 2001)

3.3.3.1.3. Grid system arrangement

In this arrangement system is like tiling the units are arranged in a grid based on rectangular or square, sometimes equilateral triangles and orthogons.



a. Grid arrangement of visual units



b. Grid arrangement of muqarnas groups (each is of radical arrangement of visual units)

Figure 3.23. *Grid Arrangement (Yaghan, 2001)*

3.3.3.1.4. Composite system arrangement

This arrangement is a composition of two or more as shown in the second ‘b’ figure of the three arrangements. After the full development of muqarnas, which can be said 13th century, they are of composite arrangements showing the advancement and complexity of geometric patterns in the Muslim world.

The evolution is seen in the table below as from selected exemplary buildings up to 14th century works with how the layering and 2DPP has advanced followed by the complexity of arrangement of the Muqarnas.

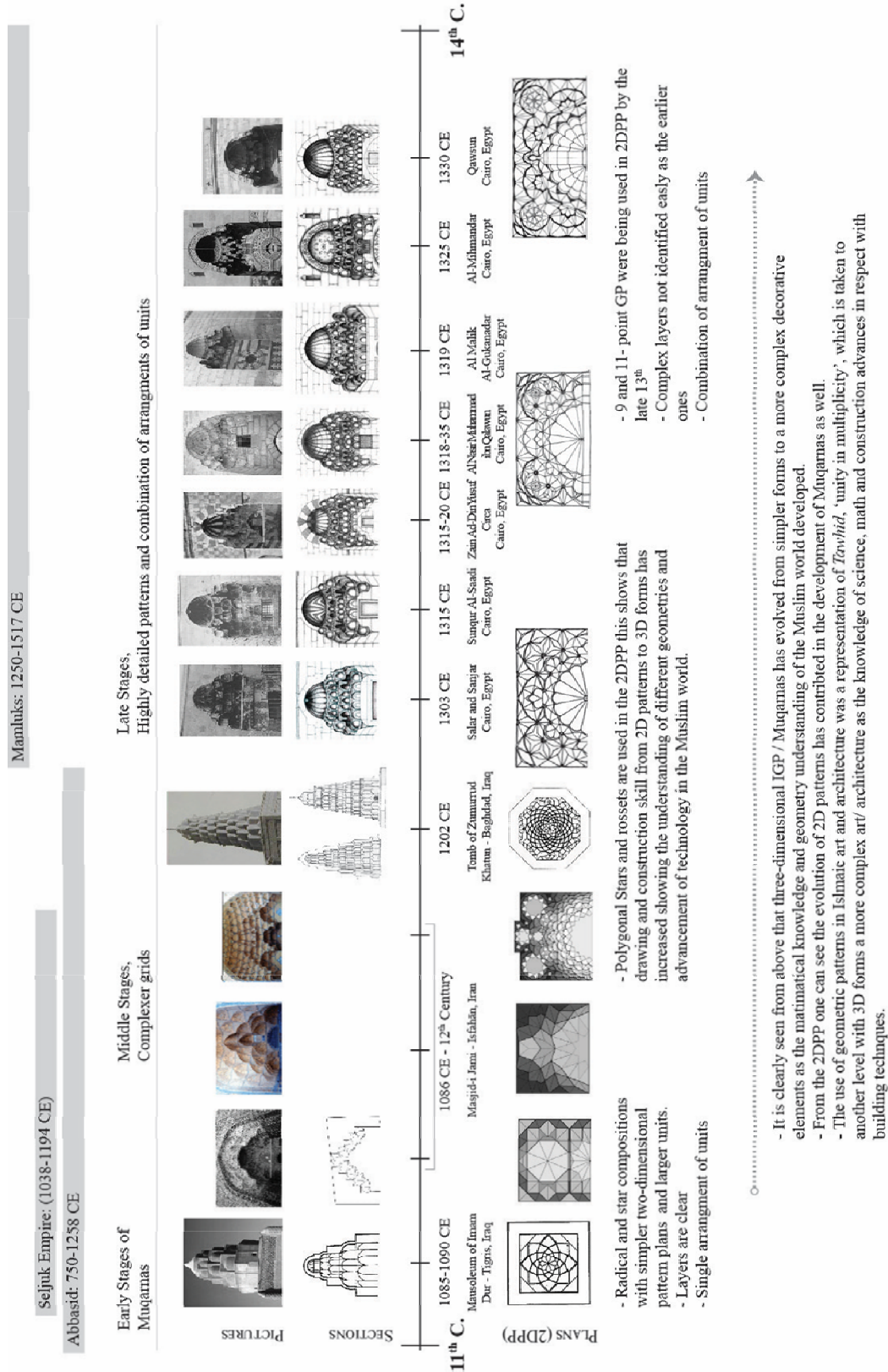


Figure 3.24. Summary of evolution of three-dimensional Islamic geometric patterns from 11th to 14th century

Yaghan argues that the Modern Islamic architecture has three attitudes towards muqarnas; “total neglect, direct copy from the past, and the use of very simplified versions of form” (Yaghan, Muqarnas, 2001, p. 8). This is due to two factors. First, the difficulty in producing such a form manually, or even by using computer drafted system and second, the lack of the proper knowledge of the process of muqarnas creation and its geometry. Therefore, he breaks it down into composition of elements to give it a potential definition as a theory with many variable algorithms for potential new designs developments, instead of just copy pasting from older buildings with minor changes.

3.3.4. The Elements of Muqarnas

The breaking down of Muqarnas is difficult. Yaghan explain the reason since, “there are no clear consistent definitions of the different issues of muqarnas, nor there is a consistent terminology” (Yaghan, Muqarnas, 2001, p. 8). The basic part of muqarnas are as shown in Figure below. These parts can be important changing lines and shapes of the composition of muqarnas as elements of the basic design.

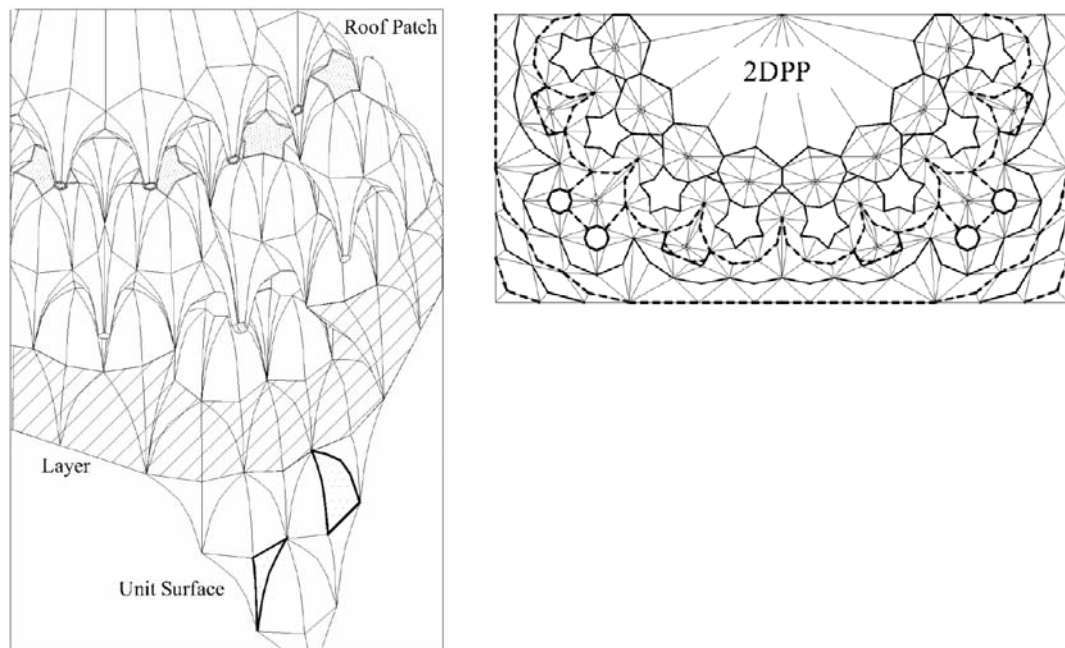


Figure 3.25. *The basic parts of Muqarnas (Yaghan, 2010)*

3.3.4.1. Units

Unit is the series of pieces that the muqarnas is composed of. There are levels of units also named Unit-surface and Visual unit. Unit-surface is “a three-dimensional surface represented in plan either by a single line or a compartment (or part of compartment) of the 2DPP”, (Yaghan, 2001, p. 34).

Unit surface are further categorized as edge to point (ETP), point to edge (PTE) and edge to edge (ETE) based on their presence as shown in the figure below. Visual unit is one or a group of same type unit-surfaces connected via their side edges.

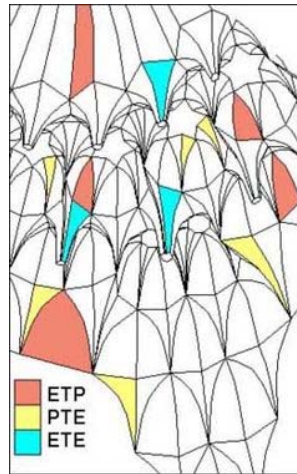


Figure 3.26. Unit surface, ETP, PTE and ETE, (<http://muqarnas.muqarnas.org/muq-course/overview.html>)

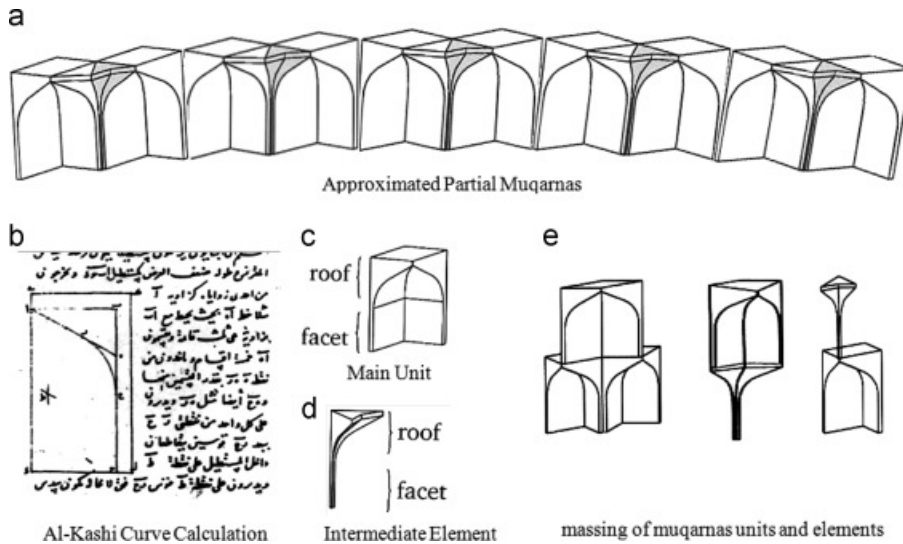


Figure 3.27. Visual unit and Erection units of the muqarnas (Kashef, 2017)

The changeable variable in the unit of muqarnas is the curvature and that comes from the change in dimension of the unit height and the vertical and horizontal joints.

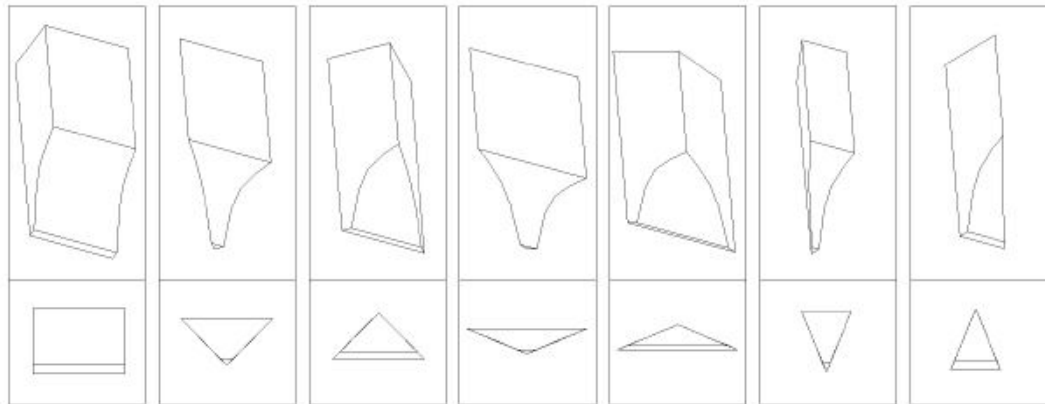


Figure 3.28. The plans and forms of generic group of muqarnas units according to (Yaghan, 2010, p. 116)

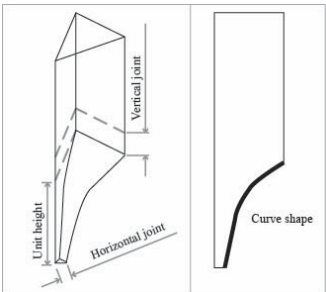


Figure 3.29. The basic variables of a unit (Yaghan, 2010)

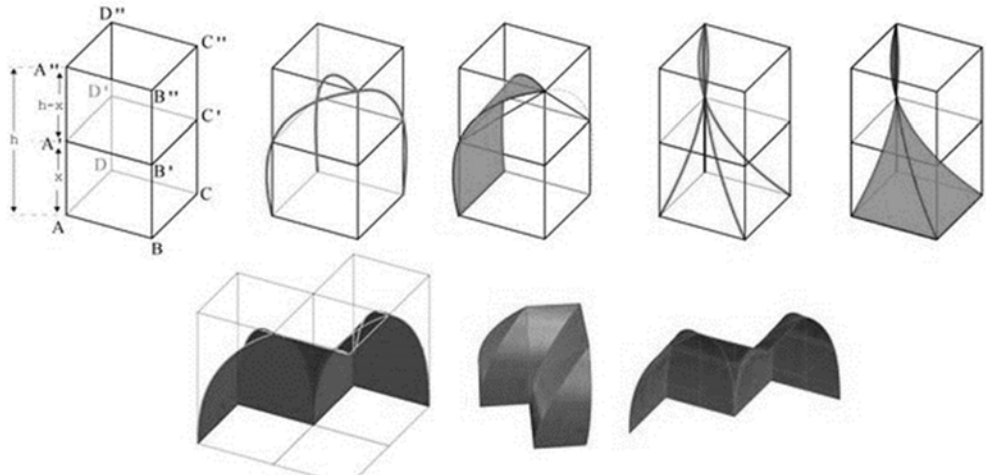


Figure 3.30. Modelling exercise and type of a muqarnas unit (Alaçam & Güzelci, 2016)

3.3.4.2. Two-Dimensional pattern plans

Two-Dimensional patterns plan, the plan organizing system for the units and the layers of the muqarnas. To understand the 2DPP and their design changes and flexible variables has been discussed above as Two-dimensional Islamic geometric patterns and their evolvement.

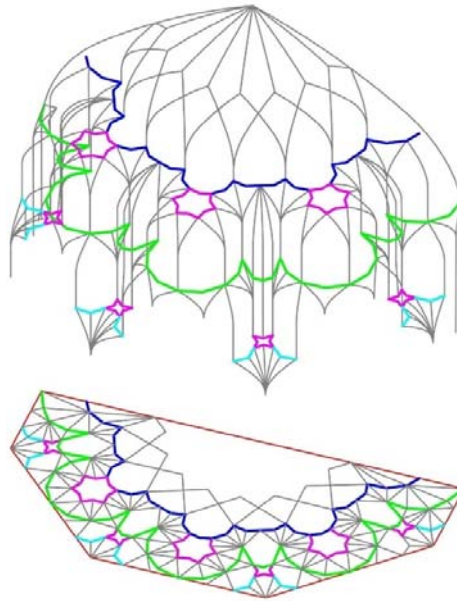


Figure 3.31. 2DPP of simple Muqarnas design drawing by Ali Reza Sarvdalir

3.3.4.3. Tier or Layers

Tiers are each of the successive levels or layers in which the constituent elements of muqarnas are arranged.

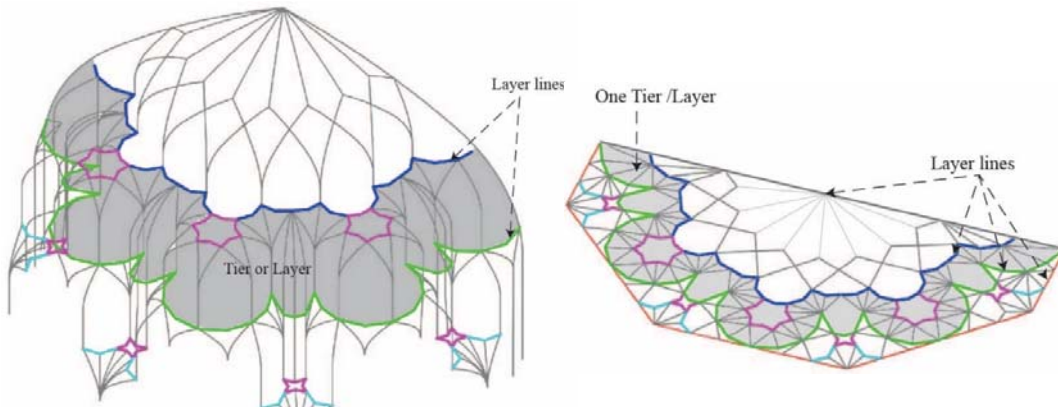


Figure 3.32. Three layered muqarnas including the top dome and lower partial units with four-layer lines including the lowest line and the top point.

From this identification of basic elements Yaghan has given the computer-generated program to generating new forms. By changing the defining lines of the elements in the unit and layer he has showed the potential new forms that can be generated.

The definition of elements and alteration techniques given above are from a literature review of Yaghan and other scholars which as for my opinion are not enough breakdown elements in defining or identifying muqarnas as important pieces giving the best description of Islamic geometric pattern. The 3D patterns of Islamic architecture has used different mathematical geometries in their time to apply them in curved surfaces as of domes or squinches and other, showing the joining of the units happened in a surface that was either flat or curved which follows the laws of Euclidean geometry when it is flat and in surfaces of domes or vaults i.e. curved surfaces is using principles or postulates of spherical geometry. Meaning there is an additional element to consider not directly drawn in like the other lines but as a surface in the background which can be called as 'Geometric Surface'. The names given is for the purpose of this paper it is not an agreed upon term.

3.3.4.4. *Geometric surface*

A surface where all edges of same elevation, mostly flat or curved, are arranged. This can be considered as the back surface where the units are all joined. This surface is flat sometimes like in a grid arrangement of visual units (Figure 3.23 a.) or can also be curved like in muqarnas of Masjid-I Jami (Figure 3.20) or spherical as in dome based muqarnases mostly seen in radical arrangements. These surfaces are Euclidean geometry based when they are flat surfaces and spherical geometry when curved or radical.

4. NEW MUQARNAS DESIGNS

New forms can be created using the algorithm of elements of muqarnas discussed above. Meaning the elements of muqarnas are considered as the changeable variables to be fed in the algorithm to come up with new structural and decorative muqarnas works¹¹.

One of the challenges in Muqarnas development can be said construction, which might have been the limiting factor in the advancement of Muqarnas designs after the 16th century. The drawing of a complicated 3D geometric pattern is difficult to visualize all angles and views. Standard drawing as elevations, sections and plans run short to express and understand the complete form. Transferring this to construction is even more challenging and leads to try and error works. The visual alterations of small angles in making tessellations and complicated 2D Islamic geometric patterns is difficult by itself, transforming that into 3D form is more challenging.

In contrast, there are also design aid programs that are referred as generated system which produce the design by themselves using unit compositions with lots of alternatives for each and combine – generate all possible member of the types¹². This concept has been used by many Islamic geometry pattern expertise and designers.

The digital 3D software should be used either as generative programs or design and experiment with 3D modeling and videos. There are numbers of software available which support algorithmic modelling and can be used to design and model muqarnas. Some of them are; CATIA, Autodesk 3ds MAX, Autodesk Revit, Autodesk Dynamo, Bentley's Generative Components and Grasshopper engine which is a plug-in for Rhinoceros. Simpler and common software as Auto CAD are also important in both 2D and 3D drafting to provide accurate drawings and intricated details. The above-mentioned 3D programs have a great advantage of enabling the designer to visualize and explore the form with many alternatives. It is possible today to experiment and design new forms of muqarnas due to this existence of these computer programs

¹¹ For the mathematical detail calculation of module designs in computer generated system see chapter 7 in: (Yaghan, 2001, pp. 59-71)

¹² For detail description refer M.A. Yaghan, (1998) "Structural Genuine-Muqarnas Dome: Type definition, Unit Analysis and Computer Generation System", *Journal of King Saud University-Architecture and Planning*, vol. 10, pp. 17-52; Yaghan, *op. cit.* 2001, pp. 57-71; W.J. Mitchell (1986), *Formal Representations: A Foundation for Computer Aided Architectural Design, Environment and Planning B: Planning and Design*, 13, pp. 133-162.

3D modeling and walk-through are digital representation of the Muqarnas, for this project, enabling the designer and user or observer to walk inside and experience the scheme in an interactive mode. This progress is the result of advancement of digital world with further steps of visual-reality programs. In the muqarnas designing of 3D modeling is important to show all the details that are not possible to be represented in detail using plan view (2DPP), elevations and sections. Computerized software is important in modelling older muqarnases as well. Modelling existing muqarnases helps in the better understanding of their formation and most importantly for restoration purposes. Besides this it also helps to create a digital library of muqarnases from all over the world which is important for detail studies and researches.

The computerized digital systems in geometric design in general are of two types: self-generated system, which are programs that make the algorithm combination and generated all possible forms or designs, and the second one the designer himself has to feed all the combinations and form or design the pattern with the 3D modelling and different views. New Muqarnas designs are also based on these two types. First self-generated computer program where the designer is aware to the change the defining character given. In this case, the unit plan and layers. And the second one the designer is the one identifying the combinations with the assistance of computer programs to create new muqarnas designs.

4.1. Computerized Self-Generated New Muqarnas Designs

Self-generated system helps to provide all the possible combination of new muqarnas designs in unit and layer alterations giving the possible constant factors and with some inputs being a defined changeable feature resulting different varieties.

4.1.1. Unit plan alteration

The unit plan alteration is related with the dimension and shape of the units of muqarnas in relation with the 2DPP, which is changing the 2D IGP and exploring all possibilities. Yaghan suggests a new system of generating unit plans taken from the divisions of 2D Islamic geometric patterns $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ types. The figure below shows

new system of generating unit plans from the 2D Islamic geometric pattern proportion ratios.

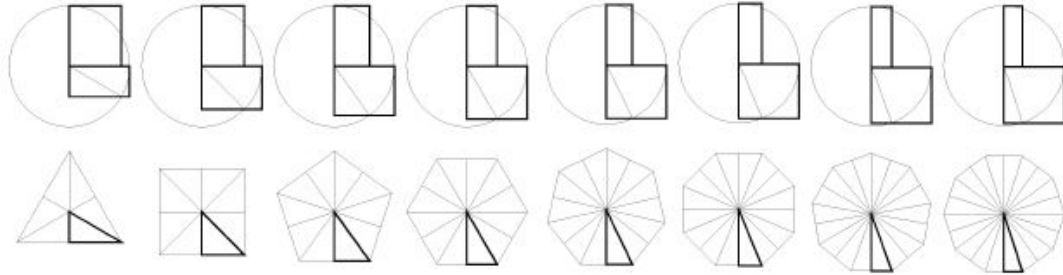


Figure 4.1. *A new system of generating pre-designed units' plan, (Yaghan, 2010)*

Example of muqarnas design formed by alteration of unit shown in Figure 5.10.

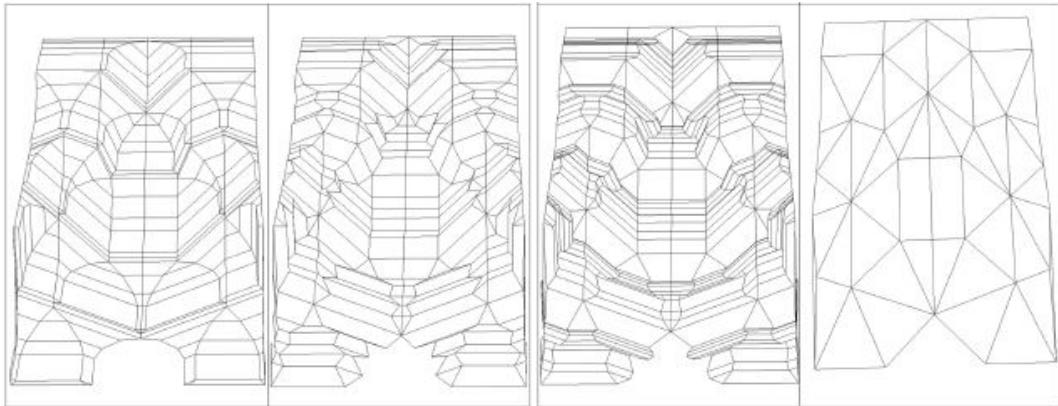


Figure 4.2. *Forms from unit alteration, the effect of the curve shape, (Yaghan, 2010)*

4.1.2. Layer alteration

In layer alteration, instead of only adding steps to the layers, imaging muqarnas in 3D form and creating splits and changing the layer lines gives a new type of muqarnas forms. This obviously requires digital 3D computer program since it is difficult to understand, experiment and design these new forms using standard drawings. Requirement of visual correction to construct the units and give a full 3D form difficult to calculate therefore, using computer generated system is a plus in new form designs of muqarnas.

Examples shown below are muqarnas forms created by changing the horizontality of the layers.

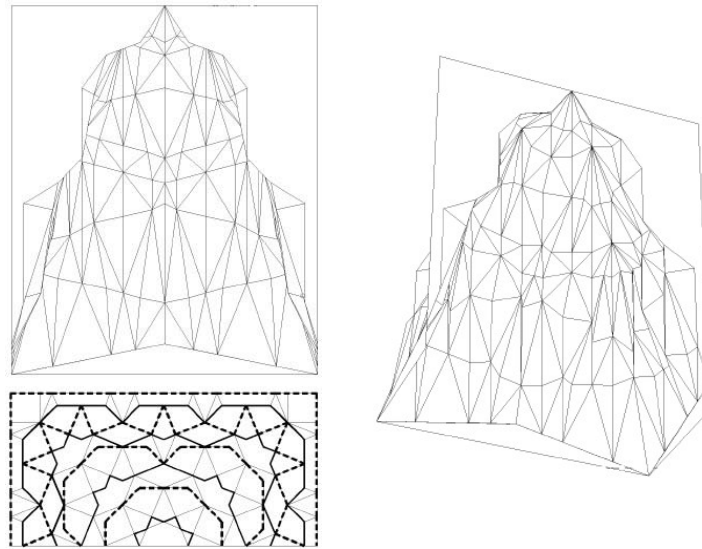


Figure 4.3. *A new muqarnas form with non-horizontal (Yaghan, 2010)*

4.2. Computer Program Assisted New Muqarnas Designs

This type is not a self-generated system where the algorithm is generated by the system, but the designer has to make the set of rules and visualize with the aid of 3D modeling software. Being able to see all details of the design before the construction in walkthroughs and virtual-reality supported programs makes the designer solve all the connection and material details beforehand. Furthermore, 3D printers had made it easier that complicate forms are printed digitally with no errors instead of being constructed manually by human labor.

4.2.1. Surface alteration

Surface alteration is changing the geometric surface that the units are attached on. Changing the geometric surface of muqarnas units from a spherical or plane surface to hyperbolic surface will give opportunity for self-supporting muqarnas structures and new type of forms.

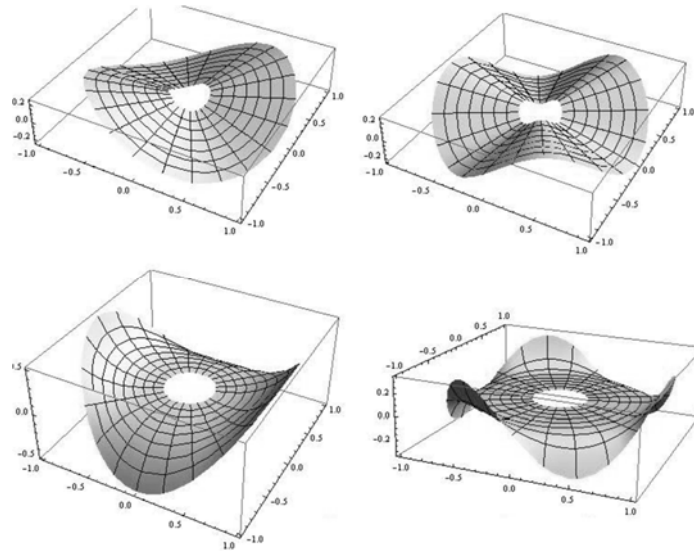
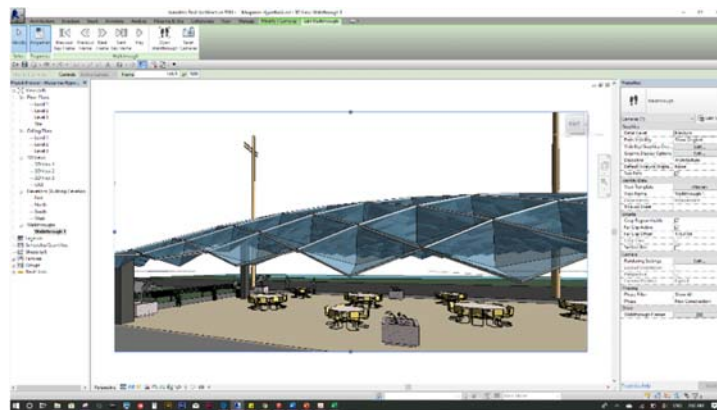


Figure 4.4. *Sample Hyperbolic geometric surfaces*

As discussed in chapter two surface alteration is dependent on the understanding of geometries. The principles of Non-Euclidean geometry are applied in changing the common linear or spherical surface arrangement of muqarnases to hyperbolic surfaces to redefine a new muqarnases typologies that can be integrated in modern building designs. Applying muqarnas units with visual correction into such a hyperbolic surface can produce both new forms and self-supporting muqarnas structures. Surface alteration is a new technique with a potential of creating various possibilities and a new point of view to muqarnas.



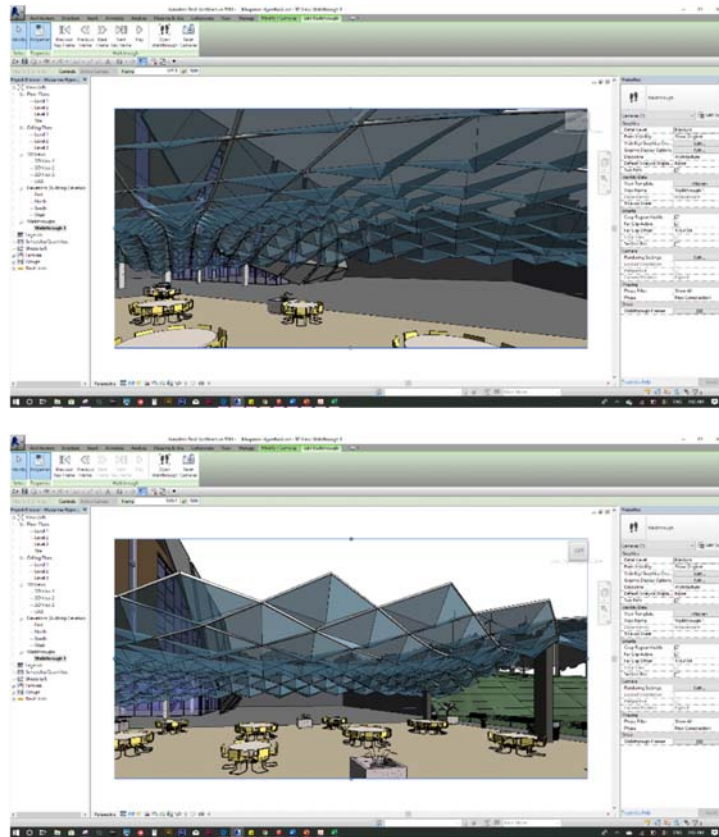


Figure 4.5. Simple Muqarnas unit on a hyperbolic surface in Revit 3D modeling software,



Figure 4.6. New Muqarnas design type with Hyperbolic surface

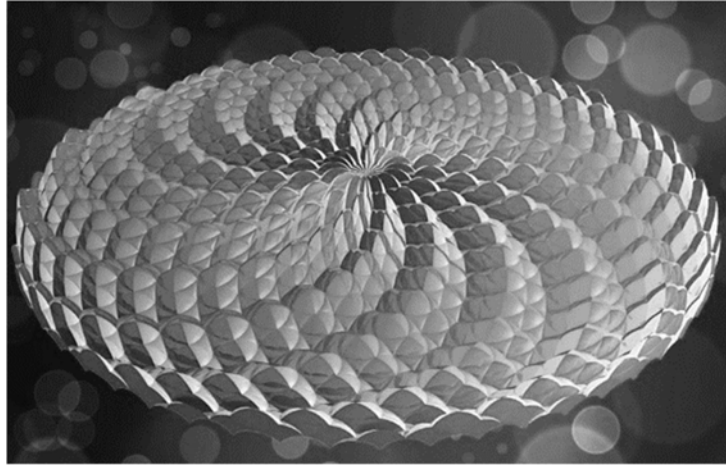


Figure 4.7. *Sample new dome/ roof structure muqarnas designs formed by changing arrangement and surface alteration (From Dan Owen's Muqarnas Designs)*

4.2.2. Unit alteration

Muqarnas given the definition is a composition of units. By changing the arrangement and composition of units new muqarnas types and definition can be formed. Yaghan has put an example in as self-supporting muqarnas by orienting the arrangement of the units focus into a line.

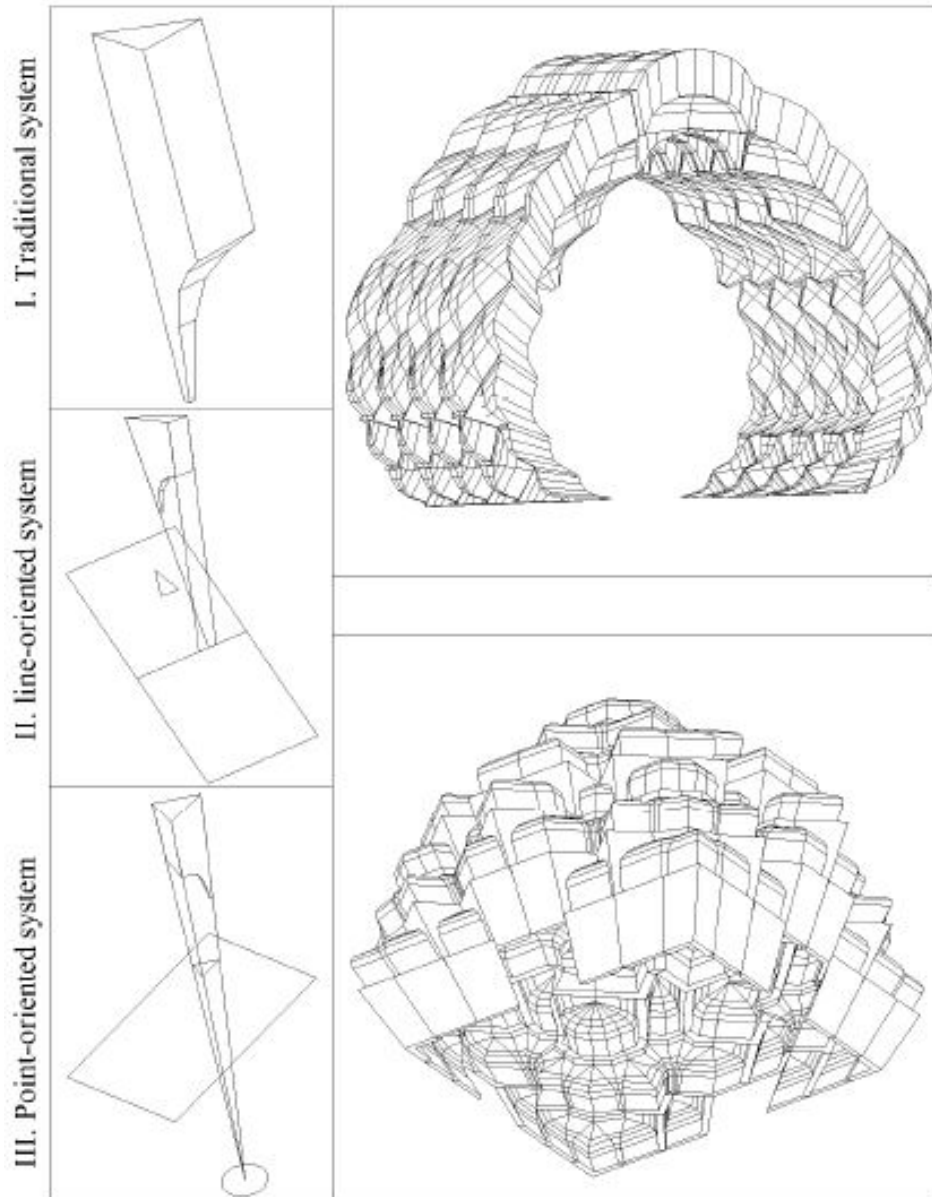


Figure 4.8. Line- and point- oriented self-supporting muqarnas (Yaghan, 2010)

It has been discussed in a lot that three-dimensional IGP are the complex developments of 2D patterns through time with a complex geometry understanding. All muqarnases are composed of units. All complex muqarnases when taken apart into pieces the basic units are the ones shown in figures 3.23, 3.24 and 3.25. Playing with this units through basic design elements and principles will give another perspective to Muqarnas

Taking only Few composed units and scaling it up to give a structure or small representation can be an example.

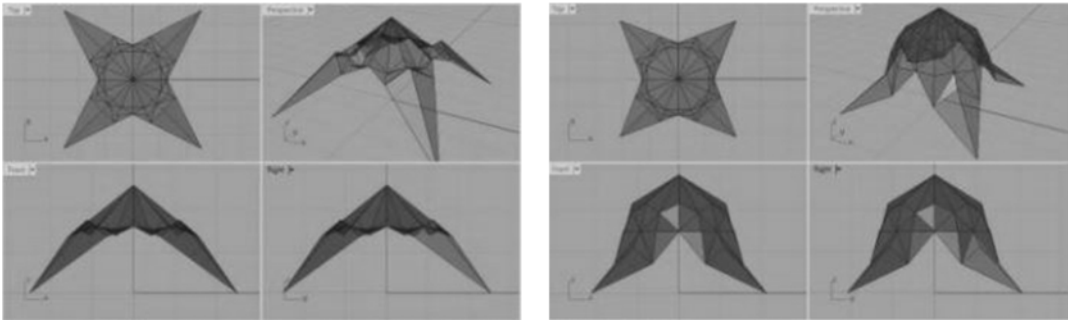


Figure 4.9. *Simple self-supporting Muqarnas structures samples 3D drawing (Sayah, 2016)*



Figure 4.10. *Muqarnas units scaled up to make self-standing structure (From the Burning Man Festival 2014, designed by Josh Haywood)*

Making this units as a skin to cover a building without losing the 3D effect, depth and basic definition of muqarnas is giving it another point of view to define it.

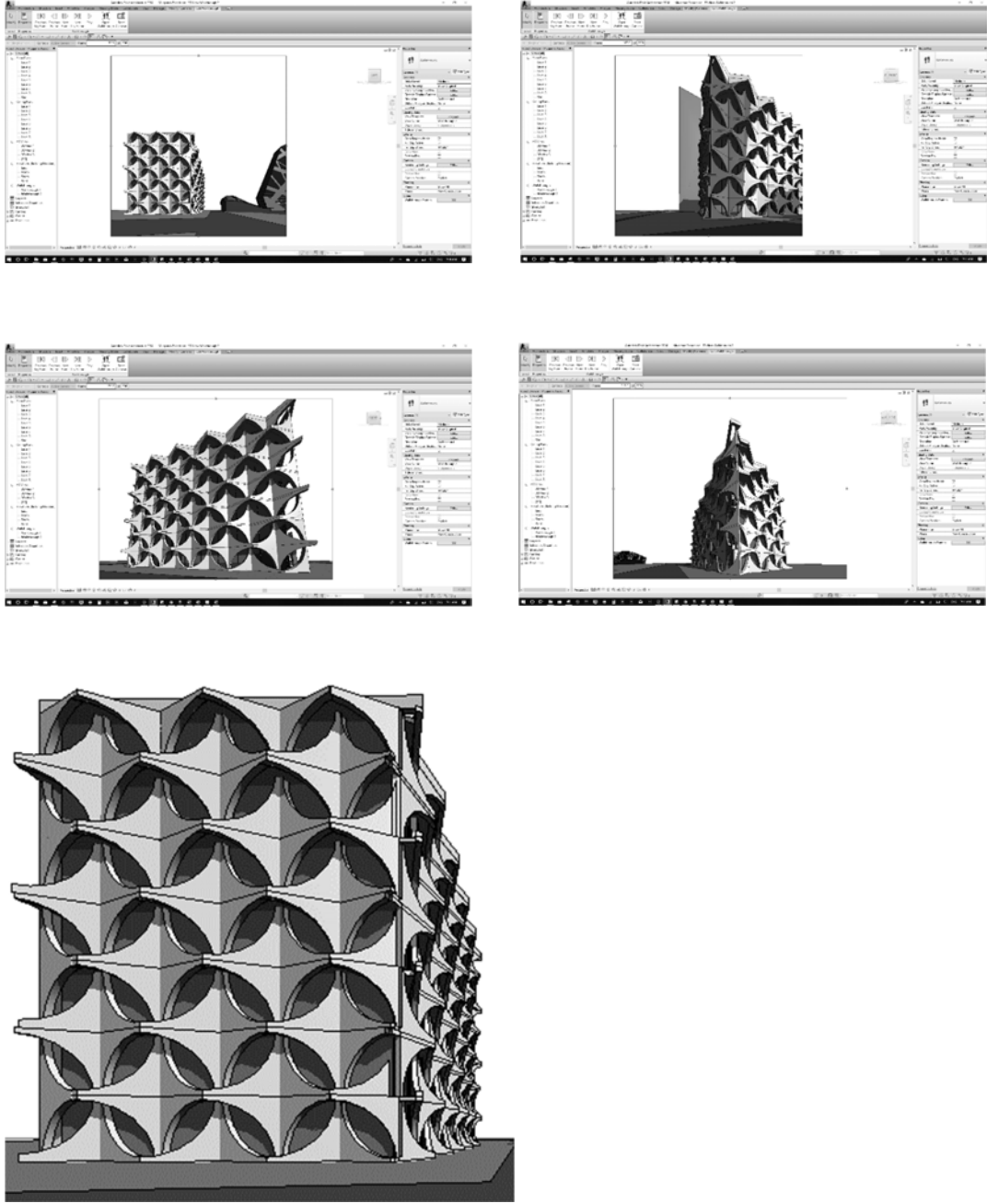


Figure 4.11. Muqarnas as a Facade

5. CONCLUSION AND RECOMMENDATION

5.1. Discussion and Conclusion

The variety of groups and nations, who under the umbrella of Islam in one as Muslims brought their various knowledge, traditions and multi-culturalism by accepting to the principles of the religion and still being able to express themselves under the given rules.

The art and architecture of the eastern world that arise after the coming of the prophet of the new religion Islam in 7th century had a major impact in the advancement of architecture in another dimension than before. What makes this art and architecture different is it shifted the narcissism of human beings who were obsessed previously with the 'Man' figure and proportion, which limits the imagination to figurative way of thinking, to an abstract and complex art based on geometry.

"From an Islamic point of view, beauty is essentially an expression of universal truth.... In excluding all anthropomorphic images, at least with in religion precincts, Islamic art helps man to be entirely himself, instead of projecting his soul outside of himself." ¹³

The significance of geometry in Islamic architecture has been mentioned earlier by scholars as Burckhardt, Kühnel, and Gayet. As Gayet's expression geometric patterns are essential characteristics of Islamic art and architecture. Both the two and three dimensional Islamic geometric patterns show the great understandings of geometry by Muslims at the time. Even though scholars like E. Gombrich concluded by saying "it is only meant to be beautiful" no need to look for other interpretations or behind meanings, others like Edith Muller provided another system of analysis using mathematical tools to study the Islamic geometric patterns claiming it is beyond ornamentation. The works of Muller are on the Alhambra of Spain concentrating on the two-dimensional patterns with some works of Muqarnas. Her work leading to the crystallographic way of studying Islamic art, which gives a clearer understanding in studying three-dimensional IGP, Muqarnas. The development and advancement of two-dimensional patterns discussed in chapters three has a great contribution to the origin of three-dimensional IGP, which can be said developed with the combination of structural elements and advancement of two-dimensional patterns with better understanding of geometry.

¹³ <http://www.studiesincomparativereligion.com/Public/default.aspx>

It is undeniable fact that geometry is the center and main element in the study of Islamic art and architecture. Islamic artwork is composed of large number of figures and units that combine to create master pieces, which is a graphical representation of *Tawhid*. Each of this module is an important part of the large design, as well as a measure of perfection and peak that allows it to be perceived as a meaningful, lively and satisfying unit by itself. They are also representation of advancement in the Muslim world in expressing Islam and the concept of *Tawhid* in an abstract way. Muqarnas are Islamic forms representing ‘unity in multiplicity’ in another level showing the understanding of math specially geometry by Muslims and their technological advancement of the time in construction.

After many arguments the it could be said that three-dimensional Islamic geometric patterns initiated somewhere in the 9th and 10th century as results of geometrical experimentation or as decorations using structural approach. Formally evolved and termed Muqarnas, becoming popular by late 11th century all over the Islamic lands.

The advancement of 2D Islamic geometric patterns led to complex 3D forms, muqarnas. 2D Islamic geometric patterns continued to develop until late 16th and beginning of 17th century. In contradiction the 3D patterns have evolved with innovations until 14th century according to Yaghan’s and early 16th century as Necipoğlu. Muqarnas has developed to its fullest by 14th century and continued minor innovations until 16th century and after continued being copied to later buildings.

After the late 17th and 18th century muqarnas works were rigid copies with no lively evolvment. Yaghan, (2001, pp. 37-38) gives the reason for this by saying:

Transformation of some fixed geometrical visual units into erection units and defining them as some sort of an alphabet of Muqarnas (which largely took place in the Islamic western lands), led eventually to a “dead end” in terms of creativity, and didn’t allow further development (which took place in the later Muqarnas of the eastern land).

This opinion is also supported by O. Grabar: “... The Alhambra stands at the end of a historical development and is, despite all its perfection, a formal dead end” (Grabar O. , 1978, pp. 181-182)

Nevertheless, Muqarnas as seen in the fourth chapter of creation of new forms has the potential of being defined by many variables as pieces of theory rather than a perfect formal dead end.

Starting with the first alterations method of unit plan, by using two-dimensional IGP with different base stars and polygons or rosettes, many variables of algorithm can be achieved. and with the second alteration, instead of adding layer one on top of the other, as the complicated works of many muqarnas works of 14th and 15th century, layer lines horizontality or angle of stacking can be altered and new techniques and forms can be produced, simple example from Yaghan's works are shown in the previous chapter.

The third and most different one is changing the geometric surface of the muqarnas unit stacks. It is discussed in chapter two about the different types of geometric planes. Applying units in hyperbolic NEG can give a new form and even definition of muqarnas.

Some of the limitations and confusions in the study of muqarnas as also mentioned by Yaghan are lack of detailed research works, documentation of drawings or scrolls as "even a section or an elevation drawings would sometimes fail in providing the actual curve design" (Yaghan, 2001, p. 48). This shows the benefit and need of computerized 3D programs in the study of Muqarnas. Technology of the 21st century provides a better solution for this. Advancements of 3d printer is one method. Printing the design directly from the computer program avoiding the understanding and explaining of construction methods to the builders will minimize the need of detail drawings to show all details of angles for construction.

A lot could be learnt from the beauty of three-dimensional Islamic geometric patterns, where skin and structure are almost one and the same. Muqarnases are transitional elements between to levels considering this into also architectural space representation it can be said they can also be a transitional medium between architectural spaces. This can be better clarified as using muqarnas as a transition from outdoor to indoor spaces as an entrance media making a gradual shift from outer to the inner areas. They can also be used to create or define a new space inside a room like stores or in defining stands or as a passage in lobbies, reception areas and so on. Summarizing this we can say muqarnas can also be used as an interior space defining component or outdoor space defining element for shelters or pergolas, or as a gradual transition form in between outdoor and indoor spaces defining it into three architectural space transitional element.

Muqarnas can be considered as designing theory or principle and be developed to be part of current works for this century as both design element and digital virtual or Alternative reality works in future advancement.

Therefore, can be defined as a three-dimensional form, whose visual function is a gradual transition between two medias and consist of units that are stacked; connected by the layer or unit corner lines.

5.2. Recommendation

Following this 21st century innovation, Islamic architecture and specially the complex works of Islamic geometric patterns which were the most complex illusion representations of the time can go a step forward in combination with visual reality, as it is the best representation of our generation, which is in need of constant change. Muqarnas new designs can be realized or represented in such a plat form to be realized and challenged to its best work by the designer and experienced by the user.

Virtual reality gives muqarnas another dimension of thinking and development. It raises the question of constantly changeable muqarnas designs in specific places that require a certain attention and significance solving the biggest problem of muqarnas construction with illusion effect of reality but no built forms. Muqarnas can be used in corridors, exhibition halls or façades of building as a constant changeable 3D effect that is experienced with the help of a visual glasses but not necessary be built with all its complicated details. Islamic geometric patterns show the original imagination capacity of the Muslim artists and advancement of community in abstract representation of art in the 11th century, which by the 21st century has a potential different representation that could be an illusion effect of inception for this generation using the technological advancement.

Another important point in the study of muqarnas is they are composition of pieces which can be broken down into elements of basic design and resigned interns of new technique using the basic design principles also. This is very important in basic design courses as a teaching method. Muqarnas is defined and taken into parts in the third chapter with line, shape, size (of units), texture, and typography which are all the basic design elements with the omitted one being color where for this research was broad and another topic by itself. Islamic geometric patterns especially muqarnas are good elements in the education system and principle for architecture and design students in the understanding of basic design principles and elements. From such a research-based teaching method can also rise a new technique and principle of muqarnas, an Islamic designing element,

contributing to the richness of both geometric based basic design teaching and to the varieties of modern muqarnases. The principle of balance and scale is done in this research as part of this paper in the fourth chapter showing the examples of new way of defining and designing muqarnas.

Color in Islamic geometric patterns are also another factor to be studied in brief as a topic which also is one of the elements making Islamic art and architectural illusional and endlessly interesting. As part of basic design element should be considered in the teaching of muqarnas and geometric patterns in Basic Design courses.

As for the knowledge of the author there is no good example to be given as a new generated muqarnas form of a built structure, though there is a good potential and researches on computer generation muqarnas forms.

For this paper is ongoing research as an advancement benefiting to the creativity and continuity of Islamic geometric patterns to be included into today's design showing the potential of contemporary usage with the technology help and simplification making it more accessible for innovations of new works and also the benefit of computerized digital systems in restoration projects. It has to be noted that the mentioned alteration factors may not be the only methods to change and develop new muqarnas form generating technique.

“Be that as it may, there has been and still is no better ambassador of good will than art.” (Ettinghausen, 1951, p. 147).

BIBLIOGRAPHY

- Abdullahi, Y., & Embi, M. R. (2013). Evolution of Islamic geometric patterns. *Frontiers of Architectural Research*, 2, 243-251.
- Alaçam, S., & Güzelci, O. (2016). Computational Interpretations of 2D Muqarnas Projections in 3D form Finiding. *ASCAAD Conference*. London.
- Al-Faruqi, I. R., & Al-Faruqi, L. L. (1986). *The culture Atlas of Islam* (4th ed.). (M. O. Kibaroglu, & Z. Kibaroglu, Trans.) Istanbul: Inkilab Press.
- Al-Shboul, R. (2017). *The modernization of Islamic art in the realm of*. Master's Theses and Doctoral Dissertations, Eastern Michigan University, Department of Interior Design, Michigan.
- Araeen, R. (2010, August 13). Preliminary Notes for the Understanding of the Historical Significance of Geometry in Arab/Islamic Thought, and its Suppressed Role in the Genealogy of World History. *Third Text*, 24(5), 509-519.
- Ardalan, N., & L., B. (1973). *The Sense of Unity: The Sufi Tradition in Persian Architecture*. Chicago: University of Chicago Press.
- Bin, M. T., & Rasdi, H. M. (2008, July 01). Reconstructing the idea of Islamic architecture: restructuring the academic framework and design approach within the perspective of the Sunnah. *The Journal of Architecture*, 13(3), 297-315. Retrieved January 2019, from <https://doi.org/10.1080/13602360802216948>
- Bonner, J. (2018, June 26). Doing th Jitterbug with Islamic Geometric Patterns. *Journal of Mathematics and the Arts*, 12(2-3), 128-143.
- Bourgion, J. (1879). *Les éléments de l'art arabe : le trait des entrelacs*. Paris: Librairie de Firmin-Didot et Cie rue Jacob, .
- Burckhardt, T. (1987). *Mirror of the Intellect: Essay on Traditionl Science and Sacred Art*. (W. Stoddart, Ed., & W. Stoddart, Trans.) New York: State University of New York Press.
- Burckhardt, T., Nasr, f. b., & Michon, i. b.-L. (First ed. 1976, Commemorative ed. 2009). *Art of Islam, Language and Meaning* (Commemorative ed.). World Wisdom, Inc.
- Chinchilla, N. L. (2007). *Converging Identities: Islamic Architecture in Detroit*. Master Thesis, University of Maryland, College Park, Department of Architecture, Detroit.
- Chorbachi, W. K. (1989). *Beyond the Symmetries of Islamic Geometric Patterns*. Cambridge: Harvard University Press.

- Cromwell, P. R. (2010). Islamic geometric designs from the Topkapı Scroll I: unusual arrangements of stars. *Journal of Mathematics and the Arts*, 4(2), 73-85.
- Dabbour, L. M. (2012). Geometric proportions: The underlying structure of design process for Islamic geometric patterns. *Frontiers of Architectural Research*, 1(4), 380-391.
- Dodds, J. D. (Ed.). (1992). *Al-Andalus: The Art of Islamic Spain*. New York: The Metropolitan Museum of Art.
- Dold-Samplonius, Y. (1992). Practical Arabic Mathematics: Measuring the Muqarnas by al-Kashi. *Centaurus*, 35, 193-242.
- Ecochard, M. (1977). *Filiation de monuments grecs, byzantins et islamiques: une question de géométrie*. Paris: Paul Geuthner.
- El-Said, I., & Parman, A. (1976). *Geometric Concepts in Islamic Art*. London: World of Islam Festival Publishing Company Ltd.
- Erdős, A., & Bettembourg, I. (Eds.). (1978). *Museum* (Vol. 30). Paris: United Nations Educational, Scientific and Cultural Organization.
- Ettinghausen, R. (1951). Islamic art and archaeology. (T. C. Young, Ed.) *Near Eastern Culture and Society*, 17-47.
- Ferreira, M. D. (2015). *Embodied Emotions: Observations and Experiments in Architecture and Corporeality*. PhD Thesis, University of Lisbon, Lisbon.
- Flood, F. B., & Necipoğlu, G. (Eds.). (2017). *A Companion to Islamic Art and Architecture* (Vol. 1 & 2). Hoboken, New Jersey, United States of America: John Wiley & Sons Inc.
- Garofalo, v. (2010). A Methodology for Studying Muqarnas: The extant Examples in Palermo. *JSTOR*, 27, 357-406.
- Golombek, L., & Wilber, D. (1988). *The Timurid Architecture of Iran and Turan* (Vol. 1). New Jersey: Princeton University Press.
- Gomez, A. P. (1999). *Architecture and The Crisis of Modern Science*. London: The MIT Press.
- Grabar, O. (1975). *The Visual Arts, in The Cambridge History of Iran : From the Arab Invasion to the Seljuqs* (Vol. 4). (R. Frye, Ed.) London: The Syndics of the Cambridge University Press.
- Grabar, O. (1978). *The Alhambra*. Cambridge, Massachusetts: Harvard University Press.
- Grabar, O. (2006). *Islamic Art and Beyond: Constructing the Study of Islamic Art* (Vol. 3). Hampshire: Ashgate Publishing Limited.

- Herzfeld, E. (1938). Arabesque. (E. Brill, Ed.) *Encyclopedia of Islam*, 1, 365-370.
- Hillenbrand, R. (1994). *Islamic architecture: Form, function, and meaning*. Edinburgh: Edinburgh University Press.
- Hoeven, S. V. (2010). *Muqarnas mathematics in Islamic Art*. Utrecht: Utrecht University Press.
- Hussain, Z. (n.d.). *Introduction to Islamic art*. Retrieved 4 10, 2019, from BBC: http://www.bbc.co.uk/religion/religions/islam/art/art_1.shtml
- Ilktürk, U. (2008). *An Attempt to Combine Mathematics and Visual Arts*. Master's Thesis, Chalmers University of Technology and University of Gothenburg, Department of Applied Information Technology, Göteborg.
- Jones, O. (1868). *The Grammar of Ornament*. London: Bernard Quaritch Piccadilly.
- Kaplan, C., & Salesin, D. (2004). Islamic Star Patterns in Absolute Geometry. *ACM Transaction of Graphics*, 23(2), 97-119.
- Kashef, M. (2017). Bahri Mamluk muqarnas portals in Egypt: Survey and analysis. *Frontiers of Architectural Research*, 1(6), 487-503.
- Kazempourfard, H. (2014). *The Evolution of Muqarnas in Iran from Pre-Seljuk to Ilkhanid Period*. PhD Thesis, Universiti Teknologi Malaysia, Faculty of Built Environment.
- Khaldun, I. (1967). *The Muqaddimah : An Introduction to History*. (N. J. Dawood, Ed., & F. Rosenthal, Trans.) New Jersey: Princeton University Press.
- Lawlor, R. (1982). *Sacred Geometry*. London: Thames Hudson Ltd.
- Makovicky, E., & Karup- Møller, S. (1977). Chemistry and Crystallography of the Lillianite Homologous series. 1. General properties and definitions. *Mineralogist Abh*, 1(130), 264-287.
- Mamedov, K. (1986). Crystallographic Patterns. *Elsevier*, 12B(3&4), 511-529.
- Mitchell, W. (1986). Formal Representations: A Foundation for Computer Aided Architectural Design. *Environment and Planning B: Planning and Design*, 13, 113-162.
- Mitchell, W. (1989). A New Agenda for Computer-Aided Architectural Design. *New Ideas and Directions for the 1990's: ACADIA Conference* (pp. 27-43). Michigan: University of Michigan.
- Mortada, H. (2003). *Traditional Islamic Principles of Built Environment*. London: Routledge.

- Necipoğlu, G. (1995). *The Topkapi Scroll - Geometry and Ornament in Islamic Architecture*. Santa Monica: The Getty Center for the History of Art and Humanities.
- Rabbat, N. (2004). Islamic Architecture as a Field of Historical Inquiry. *AD Architectural Design (Special issue Islam+Architecture)*, 74(6), 18-23.
- Rabbat, N. (2011). What is Islamic Architecture? (M. S. Jundo, Ed.) *Treasures of the Aga Khan Museum: Architecture in Islamic Arts*, 17-29.
- Reki, M., & Selçuk, S. A. (2018). Evolution of Geometric Patterns in Islamic World and a Case on the Jails of the Naulakha Pavilion in the Lahore Fort. *Gazi University Journal of Science*, 83-97.
- Riegl, A. (1992). *Problems of Style: Foundations For a History of Ornament*. Princeton: Princeton Univ. Press.
- Rosintal, J. (1928). *Pendentifs Trompes Et Stalactites Dans L'Architecture Orientale*. (P. Geuthner, Ed.) Paris: 13 Reu Jacob.
- Sarhangi, R. (Ed.). (2012). *Persian Architecture and Mathematics* (Vol. 14). Turin: Kim Williams Books.
- Sayah, I. (2016). Creating a parametric muqarnas utilizing algorithmic software. *International Journal of Review in Life Sciences*, 6(1), 47-53.
- Tabbaa, Y. (1985). The Muqarnas Dome: Its Origin and Meaning. *JSTOR*, 61-74.
- Thalal, A., Benatia, M., Jali, A., Aboufakil, Y., & Elidrissi R., M. (2011). Islamic Geometric Patterns Constructed by Craftmen Working on Wood. *Symmetry: Culture and Science*, 22, 103-130.
- UNESCO. (2011). *Gonbad-e Qābus*. Tehran: Nomination of Properties for Inscription on The World Heritage List.
- Ürey, Ö. (2010). *Use of Traditional Elements in Contemporary Mosque Architecture in Turkey*. Masters Thesis, Middle East Technical University, Ankara.
- Ürey, Ö. (2013). Transformation of Minarets in Contemporary Mosque Architecture in Turkey. *International Journal of Science Culture and Sport*, 95-107.
- Washburn, D. K., & Crowe, D. W. (1987). *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis*. Seattle: University of Washington Press.
- Yaghan, M.-A. J. (1998). Structural Genuine-Muqarnas Dome: Type definition, Unit Analysis and Computer Generation System. *Journal of King Saud University*, 10, 17-52.

- Yaghan, M.-A. J. (2001). *The Islamic Architecture Elements "Muqarnas" : Defination, Geometric Analysis, and a Computer Generated System*. Vienna, Austria: Phoibos Verlag.
- Yaghan, M.-A. J. (2010). The Evolution of architectural forms through Computer Visualization: Muqarnas example. *Electronic Visualization and the arts*, 113-120.
- Yılmaz, S. (1999). *Evolution of the Architectural Form Based on the Geometrical Concepts*. Master's Thesis, Izmir Institute of Technology, Department of Architecture, Izmir.
- Url-1 <<http://mathworld.wolfram.com/EuclidsPostulates.html>>, access date 11.05.2019.
- Url-2 <https://www-history.mcs.st-and.ac.uk/HistTopics/Euclid_definitions.html>, access date 15.05.2019
- Url-3 <<https://www.andrew.cmu.edu/user/ramesh/teaching/course/48-175/lectures/2.BasicsOfDescriptiveGeometry.pdf>>, access date 17.05.2019.
- Url-4 <<http://localcode.org/2018/03/calligraphy-beyond-pen-and-paper-in-islamic-architecture>>, access date 20.05.2019.
- Url-5 <<https://www.kisspng.com/png-floral-design-ornament-art-pattern-4738910/preview.html>>, access date 20.05.2019.
- Url-6 <https://archnet.org/sites/3872/media_contents/41008>, access date 17.05.2019.
- Url-7 <https://archnet.org/sites/10559/media_contents/111592>, access date 20.05.2019.
- Url-8 <https://archnet.org/sites/1594/media_contents/975>, access date 17.05.2019.
- Url-9 <http://archnet.org/media_contents/46235>, access date 17.05.2019.
- Url-10 <<https://www.architecturaldigest.com/story/geometric-patterns-islamic-art>>, access date 30.05.2019.
- Url-11 <<https://danielah222.wordpress.com/2015/11/19/chapter-10-the-islamic-world/>>, access date 30.05.2019.
- Url-12 <https://en.wikipedia.org/wiki/Kharragan_towers>, access date 20.05.2019.
- Url-13 <https://www.tripadvisor.com.tr/Attraction_Review-g8146493-d8124521-Reviews-Arab_Ata_Mausoleum-Tim_Samarqand_Province.html>, access date 15.05.2019.
- Url-14 <https://archnet.org/sites/1601/media_contents>, access date 15.05.2019.

- Url-15 <https://archnet.org/sites/3838/media_contents/5409>, access date 22.05.2019.
- Url-16 <<https://www.khanacademy.org/humanities/ap-art-history/west-and-central-asia/a/the-great-mosque-or-masjid-e-jameh-of-isfahan>>, access date 30.05.2019.
- Url-17 <<https://study.com/academy/lesson/hyperbolic-geometry.html>>, access date 01.06.2019.
- Url-18 <<https://altexploit.wordpress.com/2017/02/16/math-conundrum-in-thomas-pynchon/>>, access date 01.06.2019.
- Url-19 <<http://muqarnas.muqarnas.org/muq-course/overview.html>>, access date 03.06.2019.
- Url-20 <<https://archnet.org/sites/3723/publications/1369>>, access date 03.06.2019.
- Url-21 <<https://libraries.mit.edu/archives/>>, access date 03.06.2019.
- Url-22 <<http://www.studiesincomparativereligion.com/Public/default.aspx>>, access date 17.06.2019